Second order viscous corrections to the harmonic spectrum in heavy ion collisions

Li Yan

CNRS, Institut de Physique Théorique, Saclay, France

23 Oct. 2014, HENPIC Seminar

In collaboration with Derek Teaney

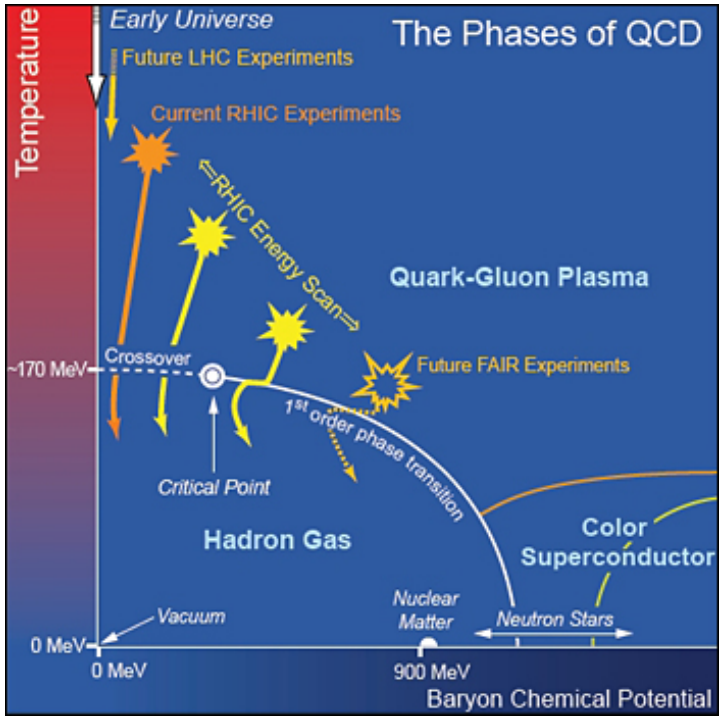
Outline

- A brief introduction to heavy-ion collisions.
- Introduction to relativistic hydrodynamics.
- Viscous corrections to distribution function at freeze-out.

  1st order viscous hydro. 2nd order viscous hydro.
- Application to calculations of harmonic flow – convergence of the calculations.
- Convergence of hydro. in a small system.
- Summary.

Introduction – QCD phase and relativistic heavy-ion collisions

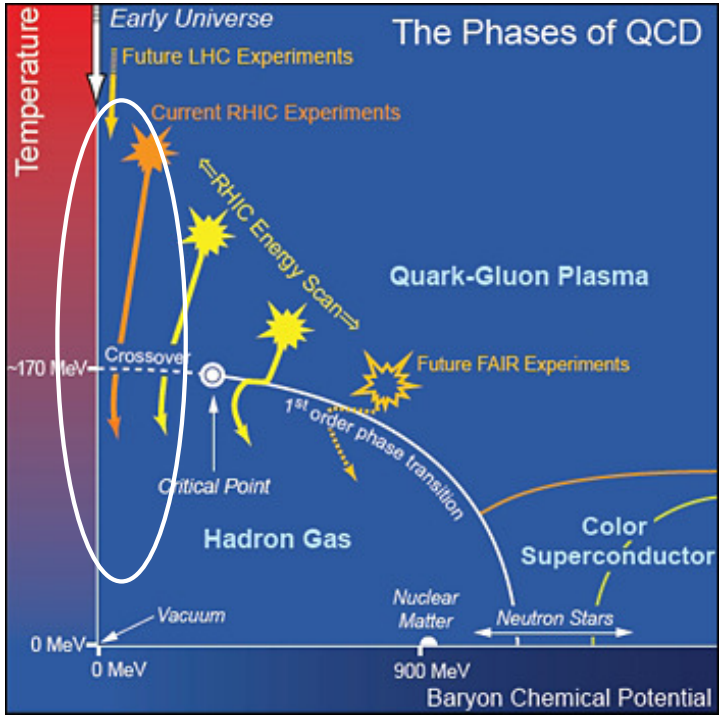
- Some facts about QCD and the QCD phase diagram:



- QCD phases : $\begin{cases} \text{Color Superconductor} & (\mu_b \gg \Lambda_{\rm QCD},\ \text{small } T) \\ \text{Quark-Gluon Plasma (QGP)} & (T \gg \Lambda_{\rm QCD}) \\ \text{Hadron Gas (HG)} & (\text{low energy scale}) \end{cases}$
- Especially, rapid crossover around $T_c \simeq 170$ MeV ($\mu_b \sim 0$) : RHIC and LHC.

Introduction – QCD phase and relativistic heavy-ion collisions

- Some facts about QCD and the QCD phase diagram:

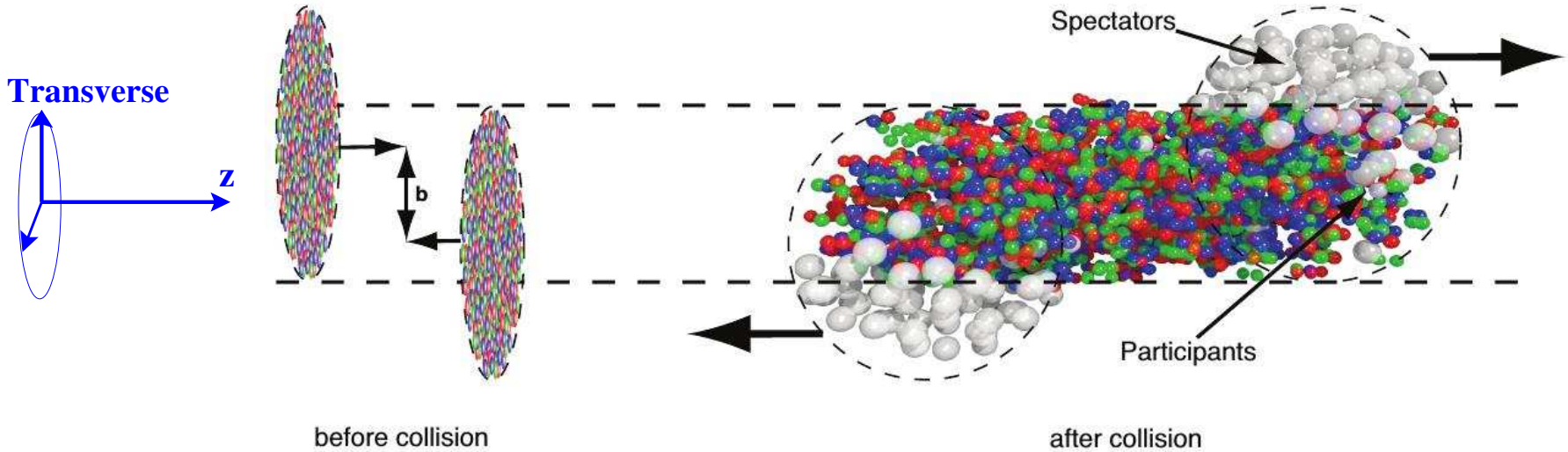


- QCD phases : $\begin{cases} \text{Color Superconductor} & (\mu_b \gg \Lambda_{\rm QCD},\ \text{small } T) \\ \text{Quark-Gluon Plasma (QGP)} & (T \gg \Lambda_{\rm QCD}) \\ \text{Hadron Gas (HG)} & (\text{low energy scale}) \end{cases}$

- Especially, rapid crossover around $T_c \simeq 170$ MeV ($\mu_b \sim 0$) : RHIC and LHC.

Large Hadron Collider

One event of heavy-ion collision



R. Snellings, arXiv:1408.1410

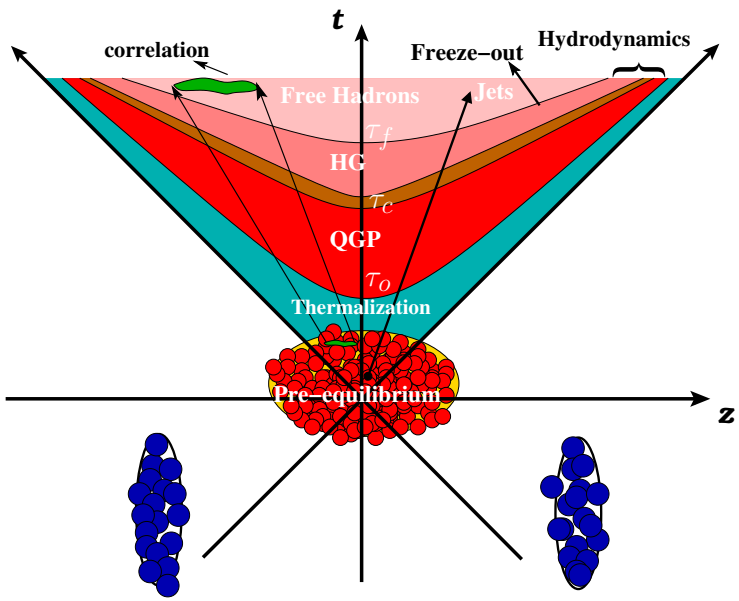
- Configuration of heavy-ion collisions:

$$\text{z-axis} \rightarrow \text{collision axis}, \quad \text{xy-plane} \rightarrow \text{transverse plane}.$$

- Number of participants: multiplicity $\sim$ centrality $\sim$ impact parameter $b \sim N_{\text{track}}^{\text{offline}}$

Introduction – QCD phase and relativistic heavy-ion collisions

- Space-time evolution of a heavy-ion collision event with QGP phase



Hard probes:

Jets, heavy flavor production, and etc.

Soft probes:

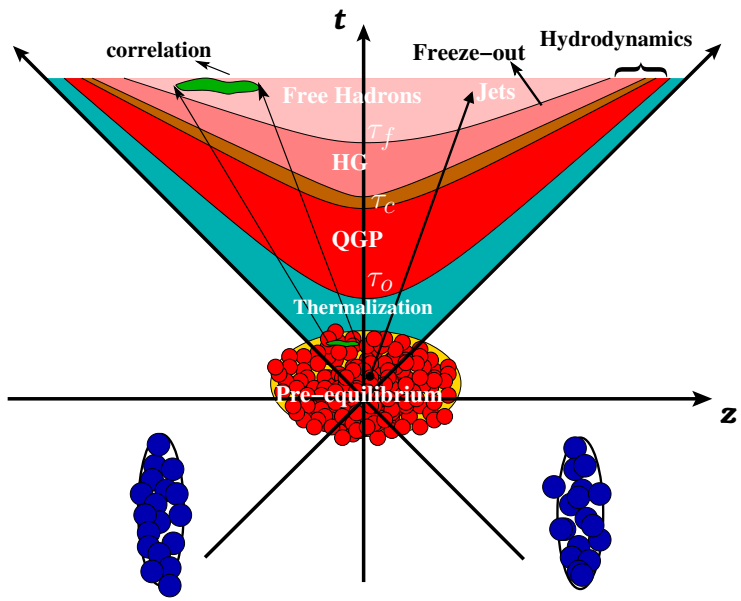
Harmonic flow, plane correlations

- Nucleus-Nucleus collision → QGP medium → Hadron Gas (HG) → Free Hadrons

- Thermalization at τ_o: Initial state models (Glauber, CGC)
- Hadronization (color confinement phase transition) at τ_c: Lattice EoS
- Kinetic freeze-out at τ_f: Cooper-Frye prescription

Introduction – QCD phase and relativistic heavy-ion collisions

- Space-time evolution of a heavy-ion collision event with QGP phase



Hard probes:

Jets, heavy flavor production, and etc.

Soft probes:

Harmonic flow, plane correlations

- Nucleus-Nucleus collision → QGP medium → Hadron Gas (HG) → Free Hadrons

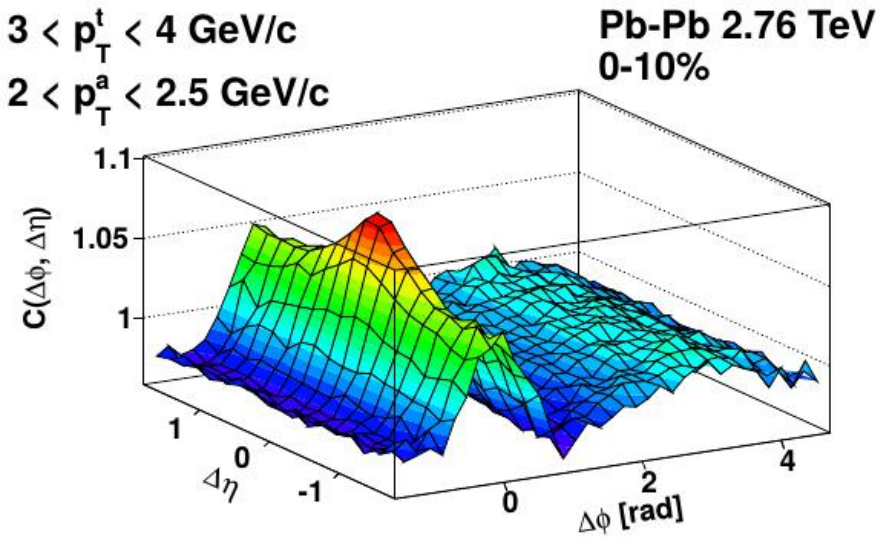
- Thermalization at τ_o: Initial state models (Glauber, CGC)
- Hadronization (color confinement phase transition) at τ_c: Lattice EoS
- Kinetic freeze-out at τ_f: Cooper-Frye prescription

$$\text{Hydro. modeling}: \underbrace{\text{initial state}}_{\text{fluc. and corr.}} \Rightarrow \underbrace{\text{hydro.}}_{\eta/s} \ldots \Rightarrow \underbrace{\text{final state observables}}_{\text{harmonic flow}}$$

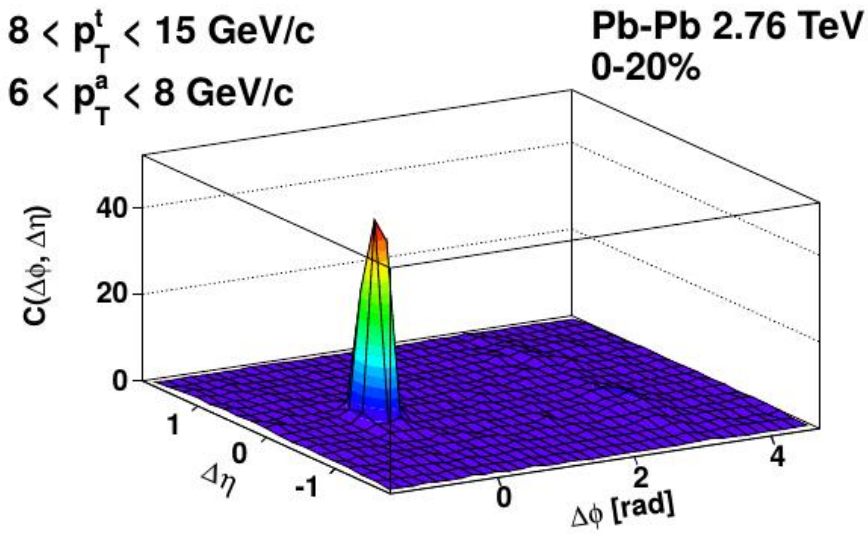
Long range correlations in heavy-ion collisions and harmonic flow

- Two-particle correlations: $(\Delta\phi_p, \Delta\eta)$ *(ALICE Collaboration, PLB 708 (2012) 249-264)*

with long-range correlation

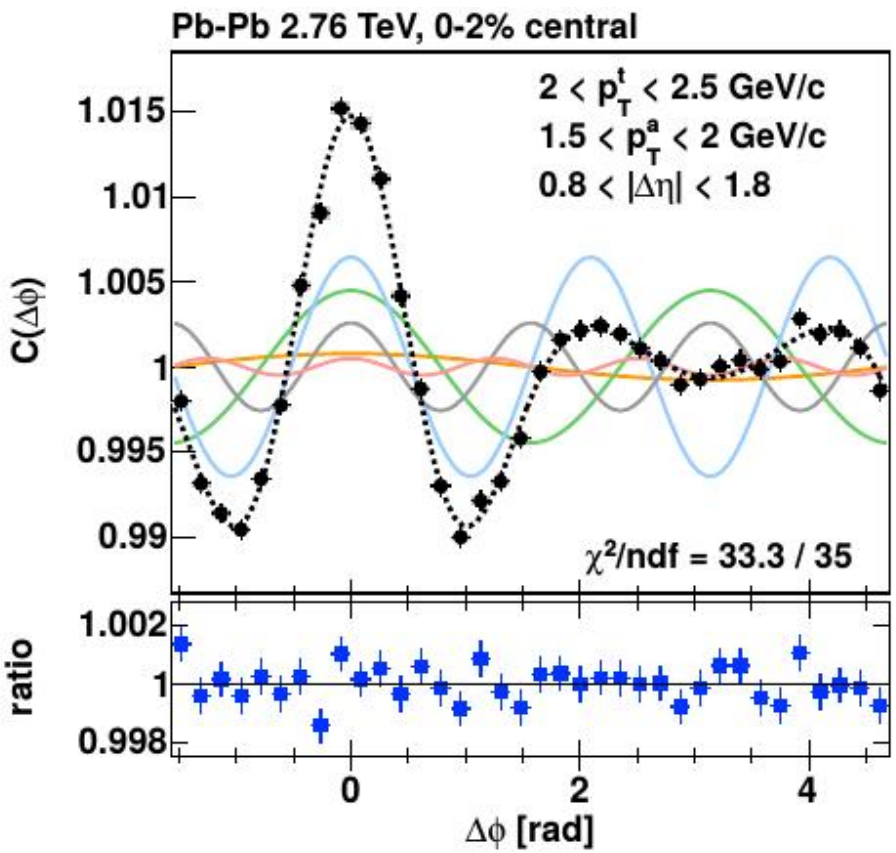


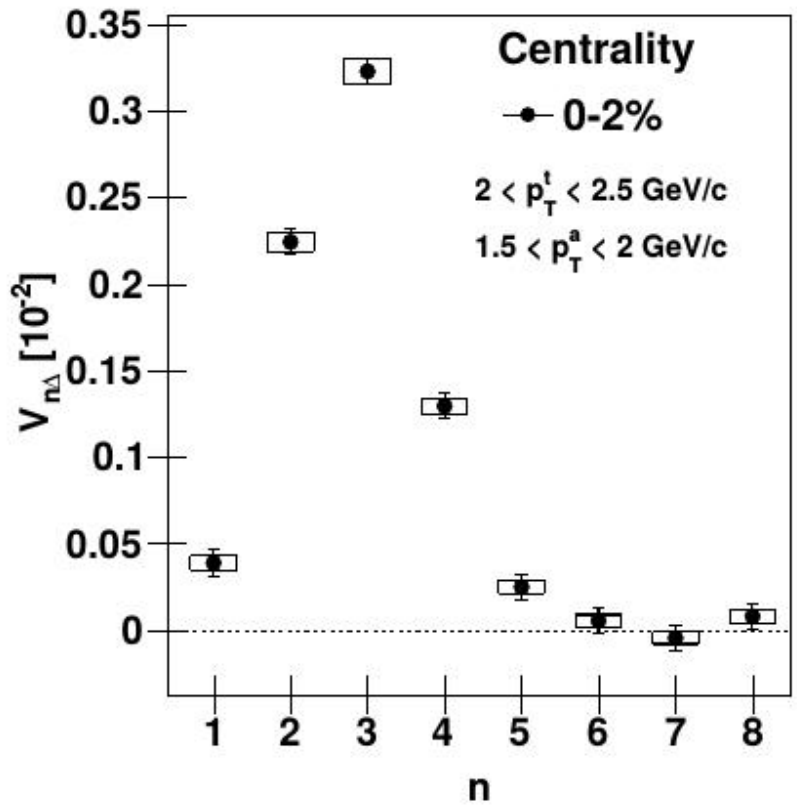
without long-range correlations



- Near-side correlation with large pseudo-rapidity range.
- Long (pseudo-rapidity) range correlation vs. $(p+p,\ A+A,\ p+A,$ multiplicity, $P_T)$

Fourier decomposition of the spectrum *(ALICE Collaboration, PLB 708 (2012) 249-264)*

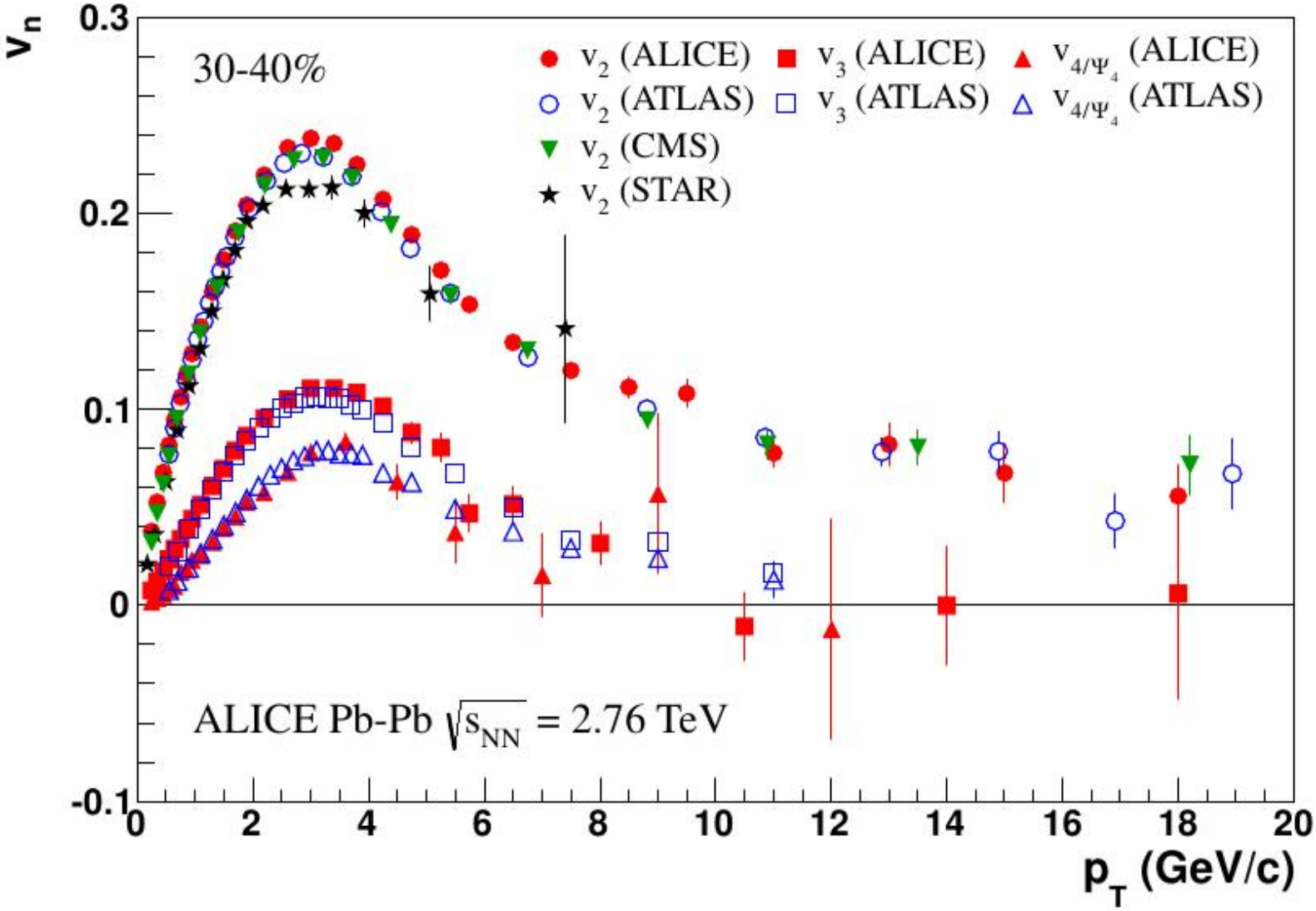




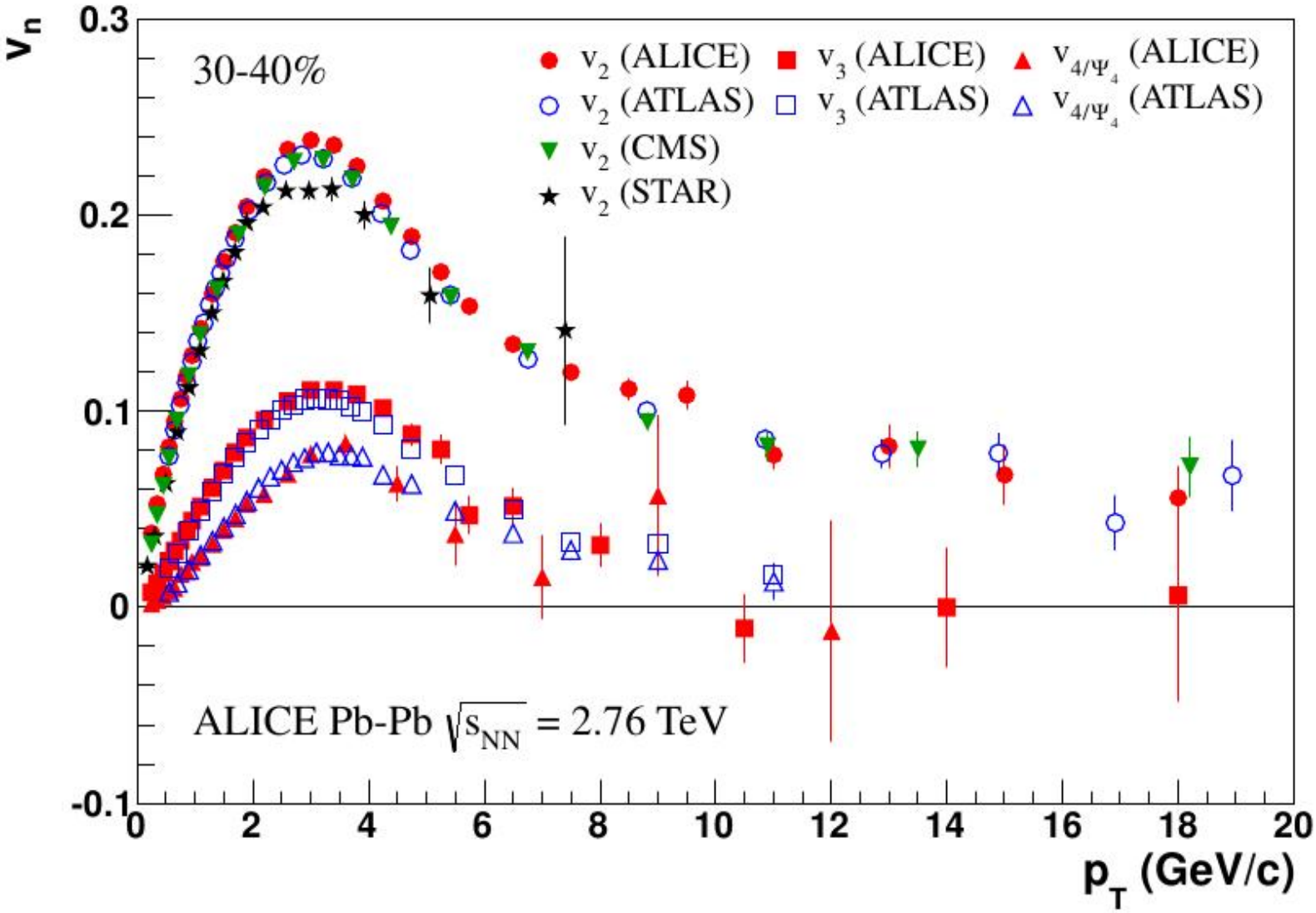
- Particle spectrum Fourier decomposition:

$$\frac{dN}{p_T dp_T d\phi_p dy} = \frac{dN}{2\pi p_T dp_T dy}\left[1 + \sum_{n=1}^{\infty} v_n(\mathrm{y}, p_T) e^{in[\phi_p - \Psi_n(\mathrm{y}, p_T)]} + c.c.\right]$$

- Harmonic flow : $V_n = v_n \exp(in\Psi_n)$ vs. (harmonic order n, multiplicity, P_T, etc.)



ALICE collaboration, PLB 719(2013)



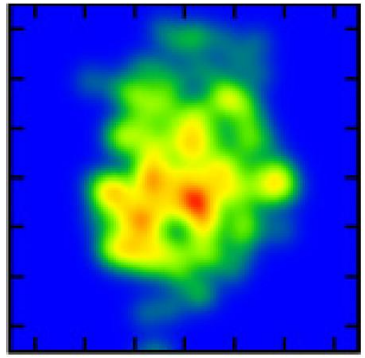
ALICE collaboration, PLB 719(2013)

Harmonic flow from experiments can be well described by viscous hydrodynamics.

Ingredients of hydrodynamic simulations for Heavy-ion collisions.

$$\boxed{\text{Initial Condition}} + \boxed{\text{Hydrodynamics}} \xrightarrow{\text{freeze-out}} \boxed{\text{Predictions}}$$

- Initial state: effectively modelled by Glauber, KLN, IP-Glasma, etc.



- Hydro. EOM: conservation of energy-momentum (and charge)

$$\partial_\mu T^{\mu\nu} = 0 \quad \& \quad (\partial_\mu n_B^\mu = 0) \quad \longrightarrow \quad u^\mu, e, \text{ and gradients of these fields}$$

- Freeze-out: generate particle spectrum from hydro.,

$$E\frac{dN}{d\mathbf{p}^3} = \frac{\nu}{(2\pi)^3}\int_\Sigma p \cdot \sigma f(x, \mathbf{p})$$

Viscous hydrodynamics and dissipative effects of the medium

- In hydro. model, dissipative corrections of higher order in the gradient expansion:

- EOM:

$$T^{\mu\nu} = eu^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \pi^{\mu\nu} \quad \Leftrightarrow \quad \begin{cases} \text{Conservation of energy-momentum. } \partial_\mu T^{\mu\nu} = 0 \\ \text{Dynamical equation of } \pi^{\mu\nu}. \end{cases}$$

1. Bulk viscosity $\zeta = 0$, so no bulk tensor.
2. Due to causality problem, 2nd order EOM ($\sim \nabla^2$) has to be considered.
3. Transport coefficients, such as η/s, are input parameters for hydro.

- Viscous corrections to the particle spectrum at freeze-out:

$$E\frac{dN}{d\mathbf{p}^3} = \frac{\nu}{(2\pi)^3}\int_\Sigma p\cdot\sigma[n(x,\mathbf{p}) + \delta f(x,\mathbf{p})]$$

where,

$$\delta f(x,\mathbf{p}) = \frac{n(1\pm n)}{2(e+\mathcal{P})T^2}p^\mu p^\nu \pi_{\mu\nu}$$

Viscous hydrodynamics and dissipative effects of the medium

- In hydro. model, dissipative corrections of higher order in the gradient expansion:

- EOM:

$$T^{\mu\nu} = eu^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \pi^{\mu\nu} \quad \Leftrightarrow \quad \begin{cases} \text{Conservation of energy-momentum. } \partial_\mu T^{\mu\nu} = 0 \\ \text{Dynamical equation of } \pi^{\mu\nu}. \end{cases}$$

1. Bulk viscosity $\zeta = 0$, so no bulk tensor.
2. Due to causality problem, 2nd order EOM ($\sim \nabla^2$) has to be considered.
3. Transport coefficients, such as η/s, are input parameters for hydro.

- Viscous corrections to the particle spectrum at freeze-out:

$$E\frac{dN}{d\mathbf{p}^3} = \frac{\nu}{(2\pi)^3}\int_\Sigma p\cdot\sigma[n(x,\mathbf{p}) + \delta f(x,\mathbf{p})]$$

where,

$$\delta f(x,\mathbf{p}) = \frac{n(1\pm n)}{2(e+\mathcal{P})T^2}p^\mu p^\nu \pi_{\mu\nu}$$

But $\delta f \sim pp\pi$ is not self-consistent in hydro. calculations.

Consistency of δf – continuity of $T^{\mu\nu}$ at freeze-out

- Kinetic theory $p \cdot \partial f = -\mathcal{C}[f] \leftrightarrow$ viscous hydro. $\partial_\mu T^{\mu\nu} = 0$

- Form of $f(x, \mathbf{p})$ is constrained as a result of consistency, (matching condition)

$$T_0^{\mu\nu} + \pi^{\mu\nu} = \nu \int \frac{d^3\mathbf{p}}{(2\pi)^3 E} p^\mu p^\nu (n(x, \mathbf{p}) + \delta f(x, \mathbf{p})),$$

- Freeze-out $\rightarrow E\frac{d^3N}{d\mathbf{p}^3} = \frac{\nu}{(2\pi)^3} \int_\Sigma p^\mu d\sigma_\mu (n(x, \mathbf{p}) + \delta f(x, \mathbf{p}))$

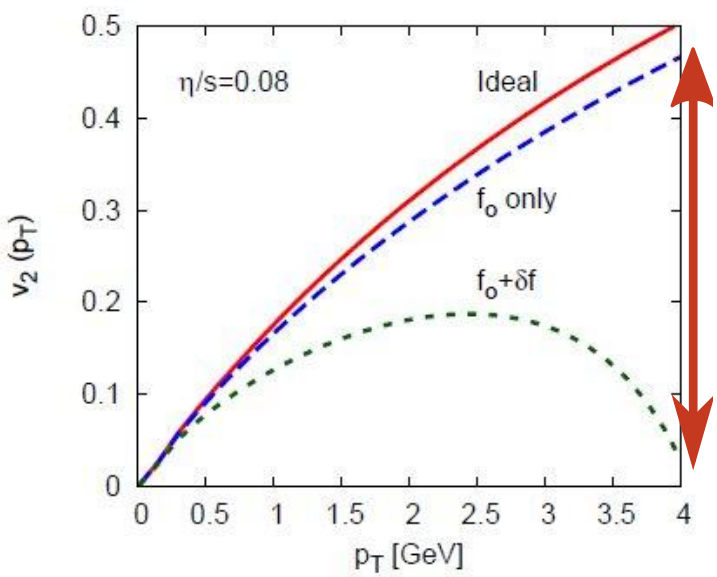
Consistency of δf – continuity of $T^{\mu\nu}$ at freeze-out

- Kinetic theory $p \cdot \partial f = -\mathcal{C}[f] \leftrightarrow$ viscous hydro. $\partial_\mu T^{\mu\nu} = 0$

- Form of $f(x, \mathbf{p})$ is constrained as a result of consistency, (matching condition)

$$T_0^{\mu\nu} + \pi^{\mu\nu} = \nu \int \frac{d^3\mathbf{p}}{(2\pi)^3 E} p^\mu p^\nu (n(x, \mathbf{p}) + \delta f(x, \mathbf{p})),$$

- Freeze-out $\rightarrow E\frac{d^3N}{d\mathbf{p}^3} = \frac{\nu}{(2\pi)^3} \int_\Sigma p^\mu d\sigma_\mu (n(x, \mathbf{p}) + \delta f(x, \mathbf{p}))$



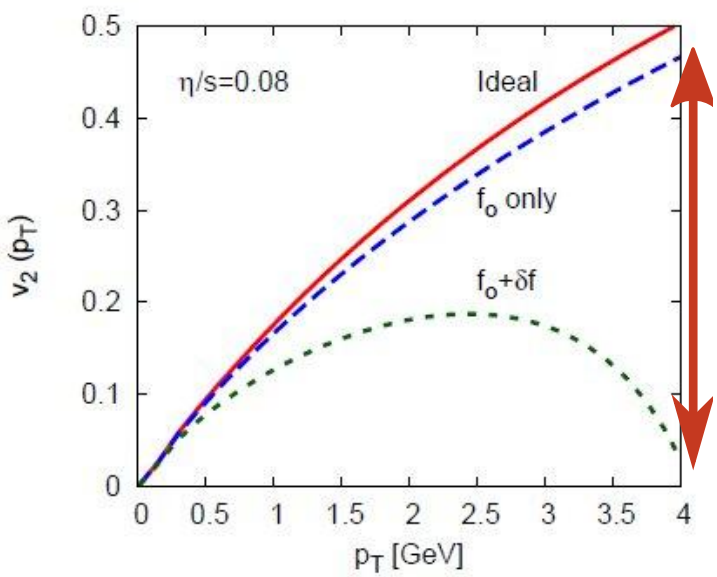
Consistency of δf – continuity of $T^{\mu\nu}$ at freeze-out

- Kinetic theory $p \cdot \partial f = -\mathcal{C}[f] \leftrightarrow$ viscous hydro. $\partial_\mu T^{\mu\nu} = 0$

- Form of $f(x, \mathbf{p})$ is constrained as a result of consistency, (matching condition)

$$T_0^{\mu\nu} + \pi^{\mu\nu} = \nu \int \frac{d^3\mathbf{p}}{(2\pi)^3 E} p^\mu p^\nu (n(x,\mathbf{p}) + \delta f(x,\mathbf{p})),$$

- Freeze-out $\rightarrow E\frac{d^3N}{d\mathbf{p}^3} = \frac{\nu}{(2\pi)^3}\int_\Sigma p^\mu d\sigma_\mu (n(x,\mathbf{p}) + \delta f(x,\mathbf{p}))$



- Most damping of flow from δf at freeze-out.
- Hydro EOM : 2nd order viscous corrections.
- Freeze-out : $f(x,\mathbf{p})$(to "1st order"),

$$f(x,\mathbf{p}) = \underbrace{n(x,\mathbf{p})}_{\text{ideal dist.}} + \underbrace{\delta f_1}_{\propto p^\mu p^\nu \pi_{\mu\nu}} + \underbrace{\delta f_2}_{?} + \ldots.$$

Determine δf for 1st order viscous hydro – (Navier-Stokes hydro.)

- Stress tensor with 1st order viscous corrections,

$$\pi^{\mu\nu} = -\eta \underbrace{2\nabla^{\langle\mu} u^{\nu\rangle}}_{\sigma^{\mu\nu}} \qquad \begin{cases} \nabla^\mu = \Delta^{\mu\nu}\partial_\nu \sim \partial^i \\ D = u^\mu \partial_\mu \sim \partial_t \\ \langle \ldots \rangle\text{: symmetric, traceless, transverse} \end{cases}$$

1. Transport equation for $f(x,\mathbf{p}) = n(x,\mathbf{p}) + \delta f_1(x,\mathbf{p}) + O(\nabla)^2$,

$$p^\mu \partial_\mu n(x,\mathbf{p}) = -\mathcal{C}[f(x,\mathbf{p})] \equiv \frac{(p\cdot u)\delta f_1}{\tau_R} = -\frac{T^2 \delta f_1}{\tilde{C}_R}$$

- Eqilibrium distribution $n(x,\mathbf{p}) = 1/(\exp(-p\cdot u/T) - 1)$.
- Relaxation time approximation $\tau_R \propto (E_p)^{1-\alpha}$ *(K.Dusling et al, PRC81, 034907(2010))*

$$\tilde{C}_R = c_p (\tilde{p}\cdot u)^{-\alpha} \qquad \begin{cases} \alpha = 0\text{: quadratic ansatz.} \\ \alpha = 1\text{: linear ansatz} \end{cases}$$

- Coefficient c_p left unknown.

2. 0th order (ideal) hydro eom,

$$D\varepsilon = -(\varepsilon + P)\nabla \cdot u + O(\nabla)$$
$$Du^\nu = -\frac{\nabla^\nu P}{\varepsilon + P} + O(\nabla) = -\nabla^\nu \ln T + O(\nabla)$$

so,

$$p \cdot \partial n(x, \mathbf{p}) = \frac{n(1 \pm n)}{2T} p^\mu p^\nu \sigma_{\mu\nu}.$$

3. Then from transport equation, to the 1st order in ∇

$$\rightarrow \qquad \delta f_1 = -\frac{n' \tilde{C}_R}{2T} \tilde{p}^\mu \tilde{p}^\nu \sigma_{\mu\nu}$$

4. Landau matching conditions: $\pi^{(1)\mu\nu} = -\eta\sigma^{\mu\nu} = \int_p p^\mu p^\nu \delta f_1$ and $u_\mu T^{\mu\nu} = e u^\nu$

$$\rightarrow \quad \eta = \frac{\nu T^3}{15} \int \frac{d^3\tilde{p}}{(2\pi)^3 \tilde{E}} [n(1 \pm n)\tilde{C}_R](\tilde{p} \cdot u)^4 \quad \xrightarrow{\alpha=0} \quad c_p = \frac{\eta}{s}$$
$$\rightarrow \quad u^\mu = u_0^\mu \quad \text{and} \quad T = T_0$$

- Some facts of the 4-step derivation:

- δf is expanded into <u>spatial gradients</u> order by order
- Hydro. EOM converts time derivatives into spatial gradients.
- $p \cdot \partial$ in transport equ. gives rise to $\nabla + \nabla^2 + \ldots$.
- Matching between $f(x, \mathbf{p})$ and hydro $T^{\mu\nu} \to (c_p, u^\mu, T)$.
- η/s is the only input parameter (also for 2nd order).
- Conformal symmetry has been taken into account – massless single component gas.

2nd order viscous hydro. – (BRSSS hydro.)

- Stress tensor with second order viscous corrections(BRSSS) *(Baier et al. JHEP 0804(2008)100)*

$$\begin{aligned}\pi^{\mu\nu} = &- \eta\sigma^{\mu\nu} \\ &+ \eta\tau_\pi \left[{}^{\langle} D\sigma^{\mu\nu\rangle} + \frac{1}{d-1}\sigma^{\mu\nu}\nabla\cdot u \right] \\ &+ \lambda_1 \sigma_\lambda^{\langle\mu}\sigma^{\nu\rangle\lambda} + \lambda_2 \sigma_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} + \lambda_3 \Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda},\end{aligned}$$

- Conformal symmetry assumed.
- Vorticity $\Omega^{\mu\nu} = \nabla^\mu u^\nu - \nabla^\nu u^\mu$, antisymmetric tensor.
- 2nd order transport coefficients: τ_π, λ_1, λ_2 and λ_3.

- Determine the corresponding form of $\delta f_2(x, \mathbf{p})$.

1. For $f(x, \mathbf{p}) = n(x, \mathbf{p}) + \delta f_1(x, \mathbf{p}) + \delta f_2(x, \mathbf{p}) + O(\nabla)^3$,

$$p^\mu \partial_\mu [n(x, \mathbf{p}) + \delta f_1(x, \mathbf{p})] = \frac{(p \cdot u)\delta^{(2)} f(x, \mathbf{p})}{\tau_R}$$

2. 1st order hydro EOM,

$$\begin{aligned}
&\partial_\mu T^{\mu\nu} = 0 \\
&\rightarrow D\varepsilon = -(\varepsilon + P)\nabla \cdot u + \frac{\eta}{4}\sigma_{\mu\nu}\sigma^{\mu\nu} + O(\nabla^2) \\
&\rightarrow Du_\alpha = -\frac{\nabla_\alpha P}{\varepsilon + P} + \frac{u_\mu D\pi^{\mu\nu}\Delta_{\alpha\nu}}{\varepsilon + P} - \frac{\Delta_{\alpha\nu}\nabla_\mu \pi^{\mu\nu}}{\varepsilon + P} + O(\nabla^2)
\end{aligned}$$

3. Identify the terms in transport equ. w.r.t. 2nd order gradients,

$$\begin{aligned}\delta f_2 =& \frac{\eta}{s}\frac{n'\tilde{C}_R}{T^2}\tilde{p}\cdot u\left[\tilde{p}\cdot q - \frac{\tilde{p}\cdot u}{4(d-1)}\sigma^2\right] + \frac{[n'\tilde{C}_R]'\tilde{C}_R}{4T^2}(\tilde{p}^\mu\tilde{p}^\nu\sigma_{\mu\nu})^2 \\ &- \frac{3[n'\tilde{C}_R]\tilde{C}_R}{2T^2}\tilde{p}^\mu\tilde{p}^\nu\sigma_{\mu\nu}\left[\tilde{p}\cdot u\frac{\nabla\cdot u}{d-1} + \tilde{p}^\alpha\nabla_\alpha lnT\right] \\ &+ \frac{[n'\tilde{C}_R]\tilde{C}_R}{2T^2}\tilde{p}^\mu\tilde{p}^\nu\left[-\tilde{p}\cdot uD\sigma_{\mu\nu} + \tilde{p}^\alpha\nabla_\alpha\sigma_{\mu\nu}\right]\end{aligned}$$

4. Landau matching $u^\mu T_{\mu\nu} = \epsilon u_\nu$ and for δf_2,

$$\begin{aligned}\int_{\mathbf{p}} p^\mu p^\nu \delta f_2 = \pi^{(2)\mu\nu} =& \eta\tau_\pi\left[D\sigma^{\langle\mu\nu\rangle} + \frac{1}{d-1}\sigma^{\mu\nu}\nabla\cdot u\right] \\ &+ \lambda_1\sigma_\lambda^{\langle\mu}\sigma^{\nu\rangle\lambda} + \lambda_2\sigma_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} + \lambda_3\Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda},\end{aligned}$$

- 2nd order transport coefficients are determined.
- (u^μ, T) obtain 2nd order corrections, so

$$\delta f_2 = \delta f_2(\text{form above}) + \text{extra contributions}\ .$$

Straightforward from δf_2,

$$\pi^{\rho\sigma(2)}/(T^2\nu) = \sigma^{\langle\rho\lambda}\sigma^{\sigma\rangle}_{\lambda}\left[\frac{2B_2}{15(d+3)} - \frac{B_3}{15}\right] - \frac{2B_3}{15}\left\{\sigma^{\langle\rho\lambda}\Omega^{\sigma\rangle}_{\lambda} - \frac{1}{2}\left[{}^{\langle}D\sigma^{\rho\sigma\rangle} + \frac{\sigma^{\rho\sigma}}{d-1}\nabla\cdot u\right]\right\}$$

where B's are constants and depend on c_p (or η/s) and α

$$B_1 = \int_{\tilde{p}}[n'\tilde{C}_R](\tilde{p}\cdot u)^4 \sim \int_{\tilde{p}}[n'\tilde{C}_R]\tilde{p}^\rho\tilde{p}^\sigma\tilde{p}^\alpha\tilde{p}^\beta,$$

$$B_2 = \int_{\tilde{p}}[n'\tilde{C}_R]'\tilde{C}_R(\tilde{p}\cdot u)^6 \sim \int_{\tilde{p}}[n'\tilde{C}_R]'\tilde{C}_R\tilde{p}^\rho\tilde{p}^\sigma\tilde{p}^\alpha\tilde{p}^\beta\tilde{p}^\mu\tilde{p}^\nu,$$

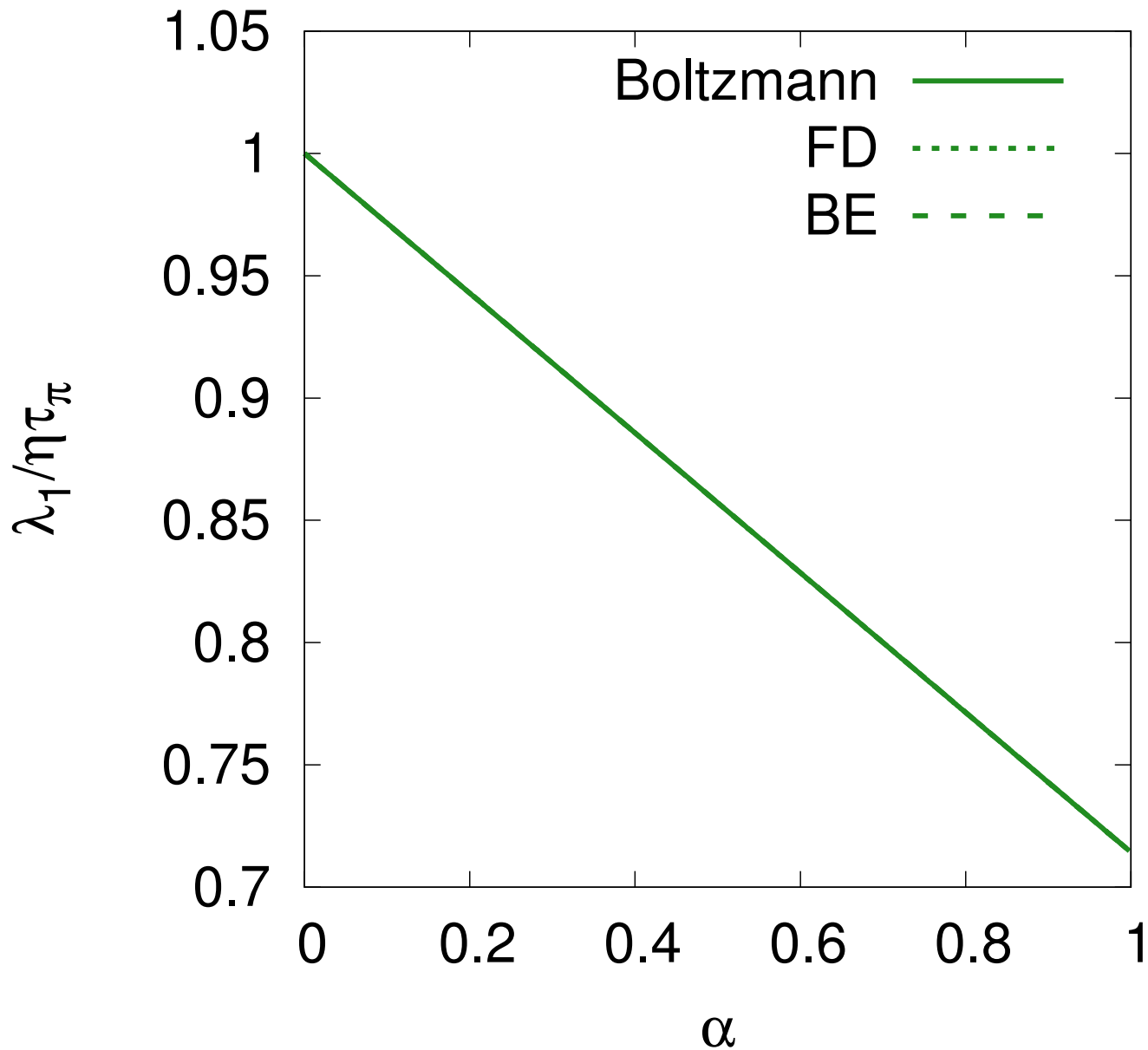
$$B_3 = -\int_{\tilde{p}}[n'\tilde{C}_R]\tilde{C}_R(\tilde{p}\cdot u)^5 \sim \int_{\tilde{p}}[n'\tilde{C}_R]\tilde{C}_R\tilde{p}^\rho\tilde{p}^\sigma\tilde{p}^\alpha\tilde{p}^\beta\tilde{p}^\mu.$$

so,

$$\lambda_1 = \left[\frac{2B_2}{15(d+3)} - \frac{B_3}{15}\right]\nu T^2, \quad \lambda_2 = -2\eta\tau_\pi$$

$$\lambda_3 = 0, \qquad \eta\tau_\pi = B_3\nu T^2/15$$

Second order transport coefficients



Implementation of $\delta f_{(2)}$ and linear harmonic flow response w_n/ϵ_n

- Linear harmonic flow response coefficient: w_n/ϵ_n

$$v_n\{2\} = \sqrt{\left(\frac{w_n}{\epsilon_n}\right)^2 \langle\langle \epsilon_n^2 \rangle\rangle}$$

- Perturbation on top of smooth Gaussian initial density $\rho(x, y)$ via cumulants $W_{n,m}$,

$$\text{e.g.: } \rho^c_{1,3} = \underbrace{\frac{\{r^3 \cos\phi_r\}}{8}}_{\sim W_{1,3}} \left[\left(\frac{\partial}{\partial x}\right)^3 + \left(\frac{\partial}{\partial y}\right)^2 \frac{\partial}{\partial x}\right] \rho_{\text{Gaussian}}(\mathbf{x}) \to \epsilon_1$$

some other cumulants: $W_{2,2} \sim \{r^2 \cos 2\phi_2\}, \quad W_{3,3} \sim \{r^3 \cos 3\phi_r\}$

(Teaney and Yan, PRC83(20011) 064904)

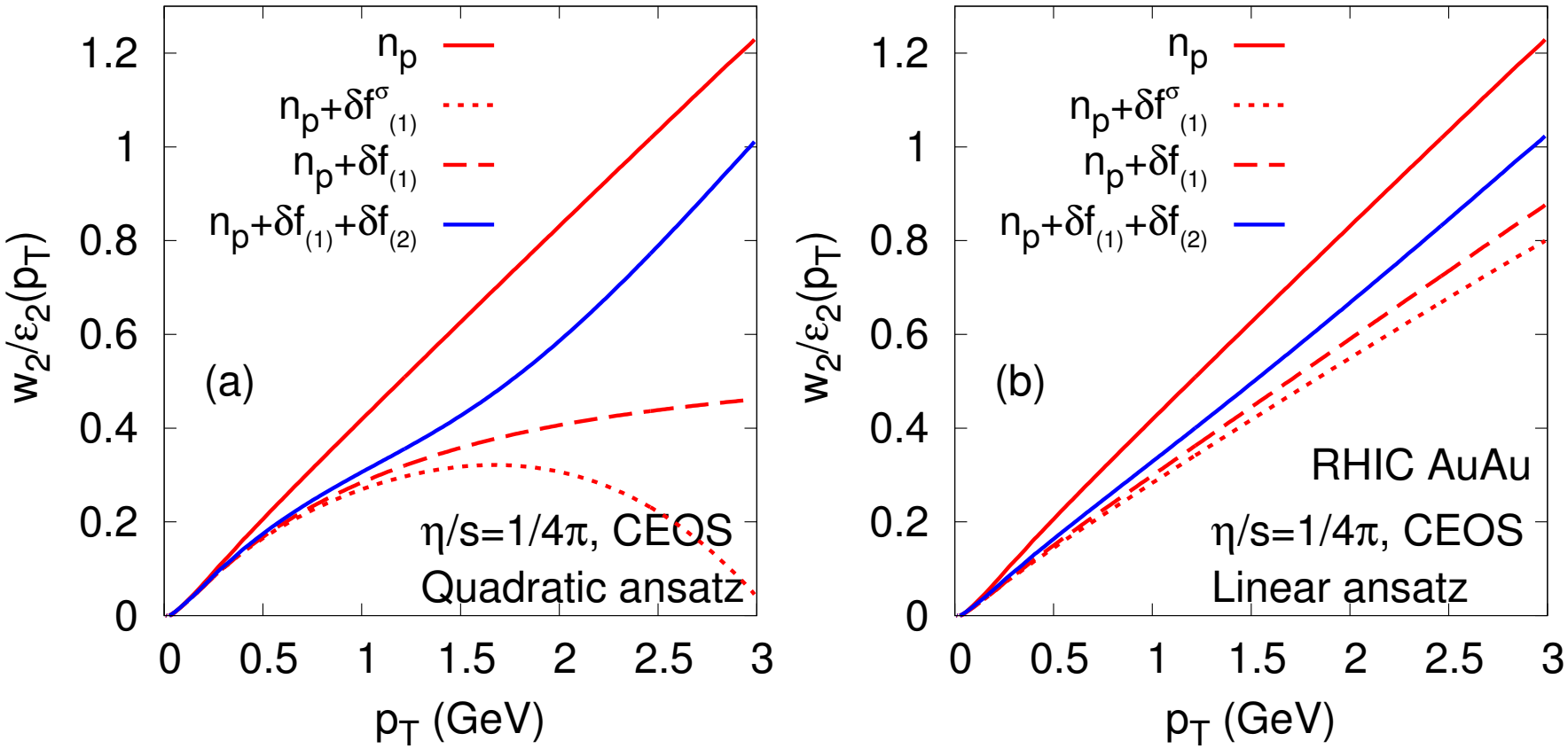
- For δf, replacing $\sigma_{\mu\nu}$ by $-\pi_{\mu\nu}/\eta$ + corrections, or just taking derivatives.

- The dependence of w_n/ϵ_n on freeze-out δf, for $b = 7.45$fm.

- n_p : equilibrium distribution n_p as the freeze-out distribution function.
- $n_p + \delta f^{\sigma}_{(1)}$: $\delta f \propto pp\sigma$.
- $n_p + \delta f_{(1)}$: $\delta f \propto pp\pi$.
- $n_p + \delta f_{(1)} + \delta f_{(2)}$: 2nd order δf.

$\delta f_{(2)}$ and elliptic flow v_2 – conformal gas

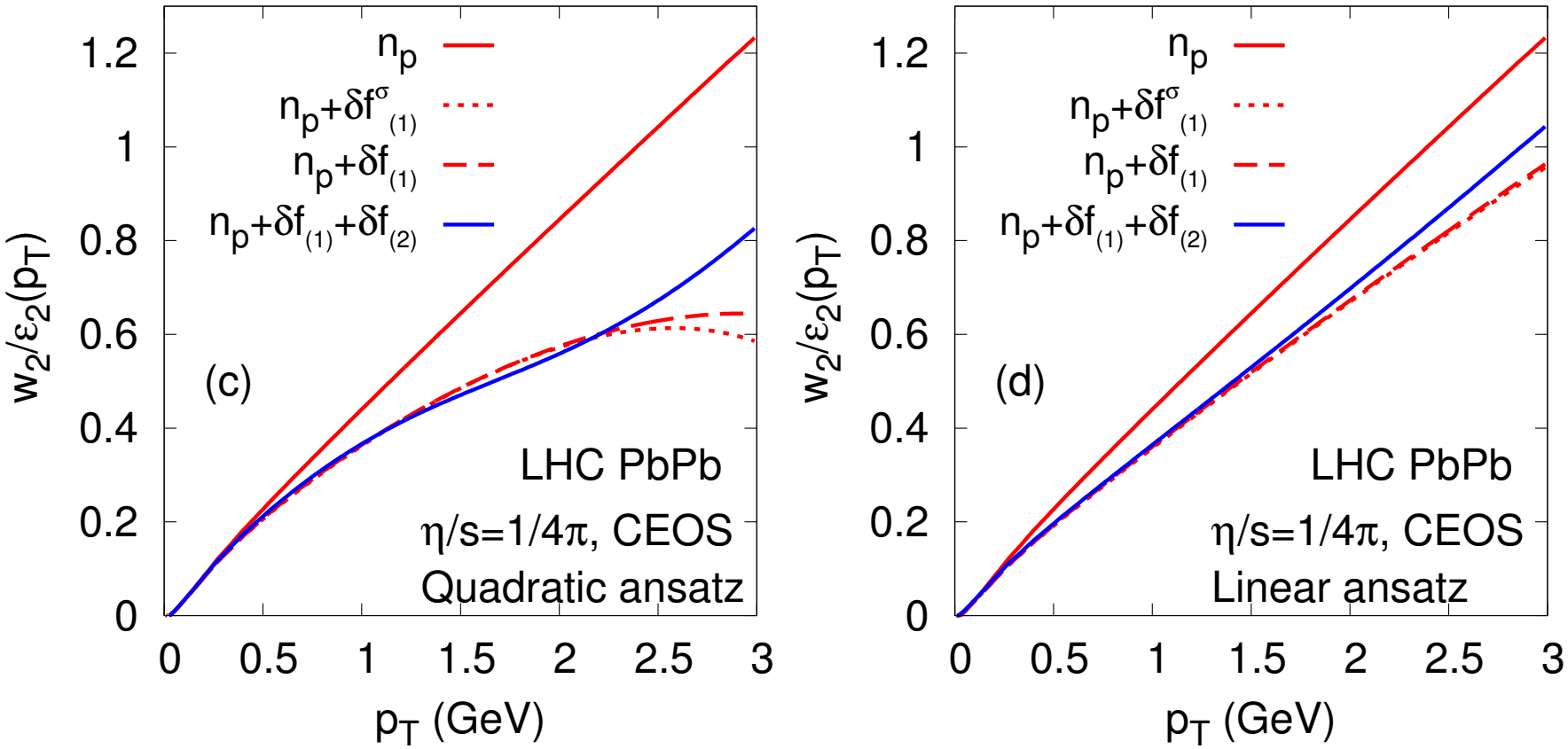
- RHIC AuAu, with conformal EOS, $\alpha = 0$ and $\alpha = 1$:



- Difference between '$f^{\sigma}_{(1)}$' and '$f_{(1)}$' indicates the magnitude of gradients.
- α affects p_T dependence and the value of transport coefficients.

$\delta f_{(2)}$ and elliptic flow v_2 – conformal gas

- LHC PbPb, with conformal EOS, $\alpha = 0$ and $\alpha = 1$:



- Difference between '$f^{\sigma}_{(1)}$' and '$f_{(1)}$' indicates the magnitude of gradients.
- α affects p_T dependence and the value of transport coefficients.

$\delta f_{(2)}$ and elliptic flow v_2 – multi-component gas

- What about non-conformal medium, with Lattice EOS?

1. Non-conformal terms are suppressed by $(\frac{1}{3} - c_s^2)$ and high p_T, for instance,

$$\delta f^{\sigma}_{(1)-\mathrm{non-conf}}(\mathbf{p}) \sim n'_p \left[\frac{p^\mu p^\nu \sigma_{\mu\nu}}{2T^3} + \left(-\frac{E_{\mathbf{p}}^2 - |\mathbf{p}|^2}{3T^3} + \frac{(\frac{1}{3} - c_s^2) E_{\mathbf{p}}^2}{T^3} \right) \nabla_\mu u^\mu \right]$$

2. For different particle species a, approximately

$$\eta/s = \frac{\sum_a \eta_a}{\sum_a s_a} = \eta_a/s_a$$

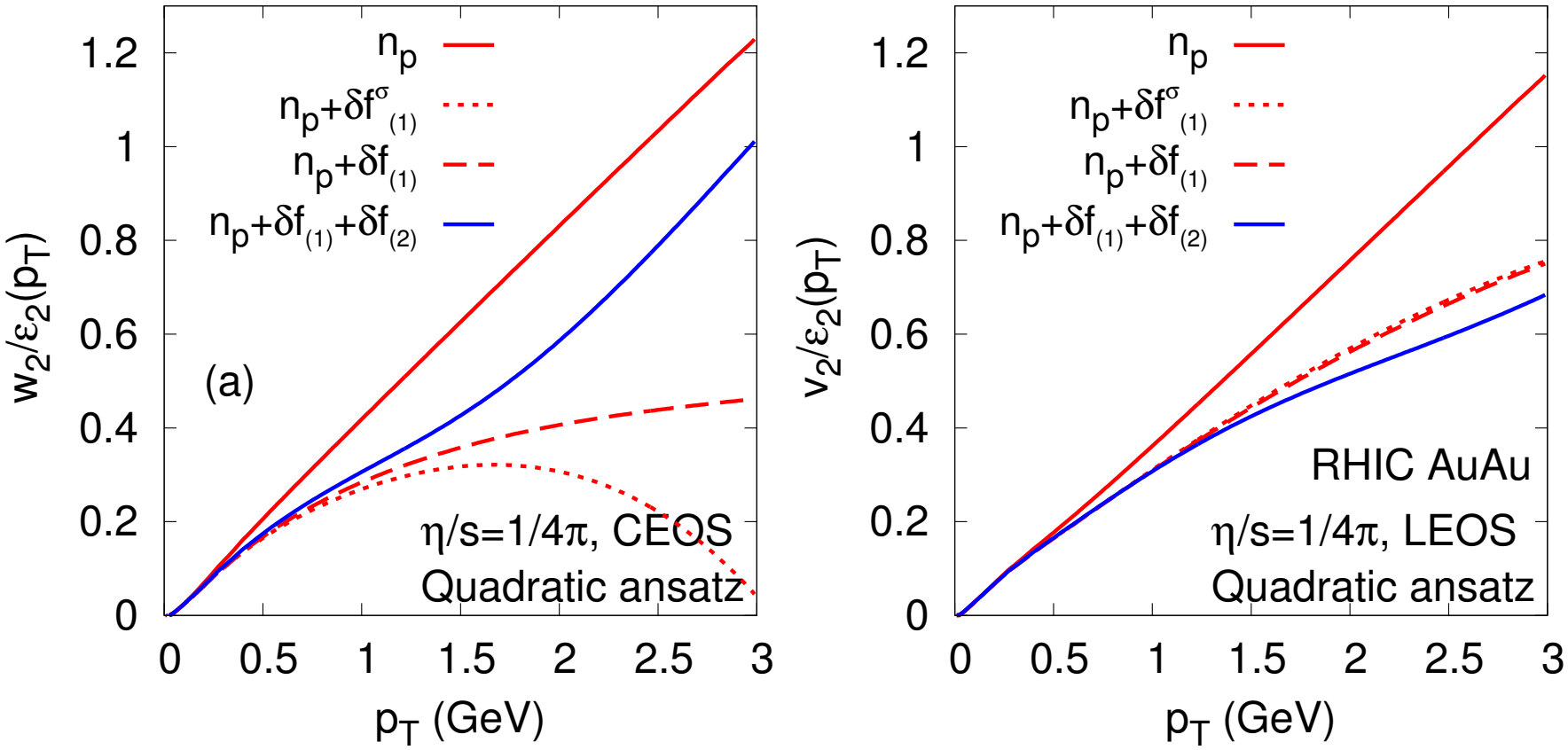
and

$$T^{\mu\nu} = \sum_a \int_p p^\mu p^\nu f$$

so for a multi-component gas the derivations are still valid.

$\delta f_{(2)}$ and elliptic flow v_2

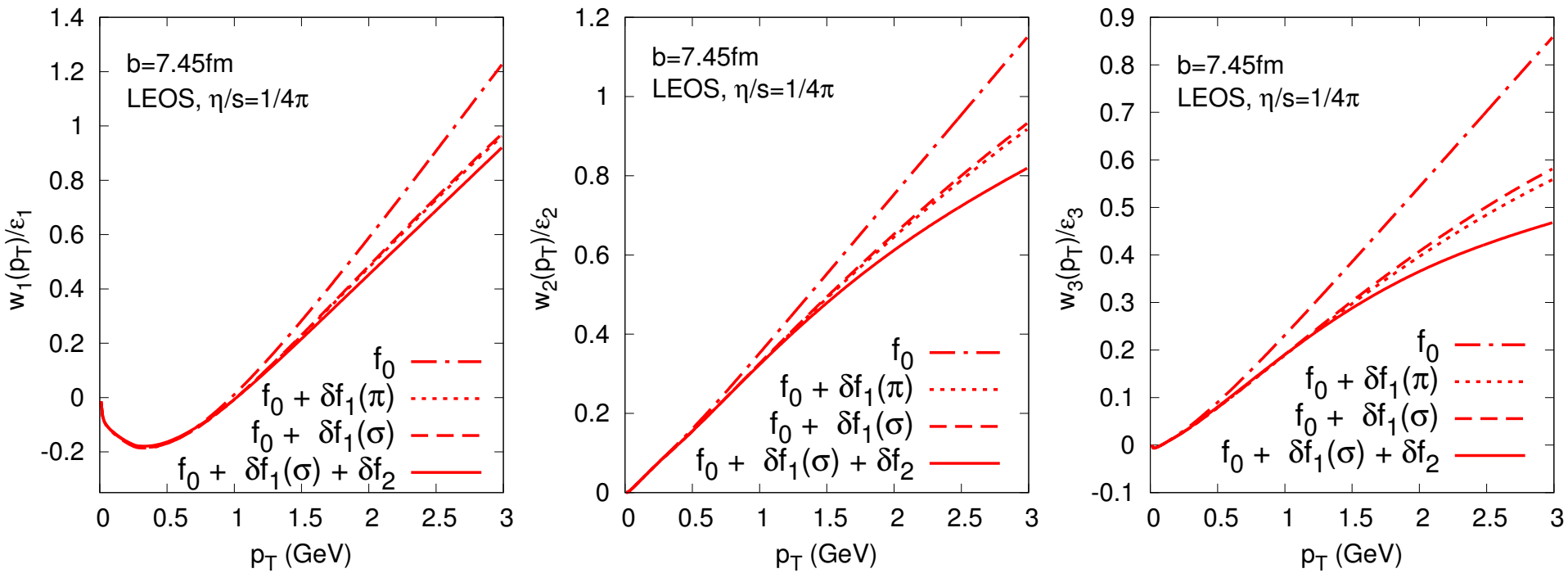
- RHIC AuAu, for $\alpha = 0$ with conformal EOS and Lattice EOS:



$\delta f_{(2)}$ and harmonic flow v_n

LHC PbPb:

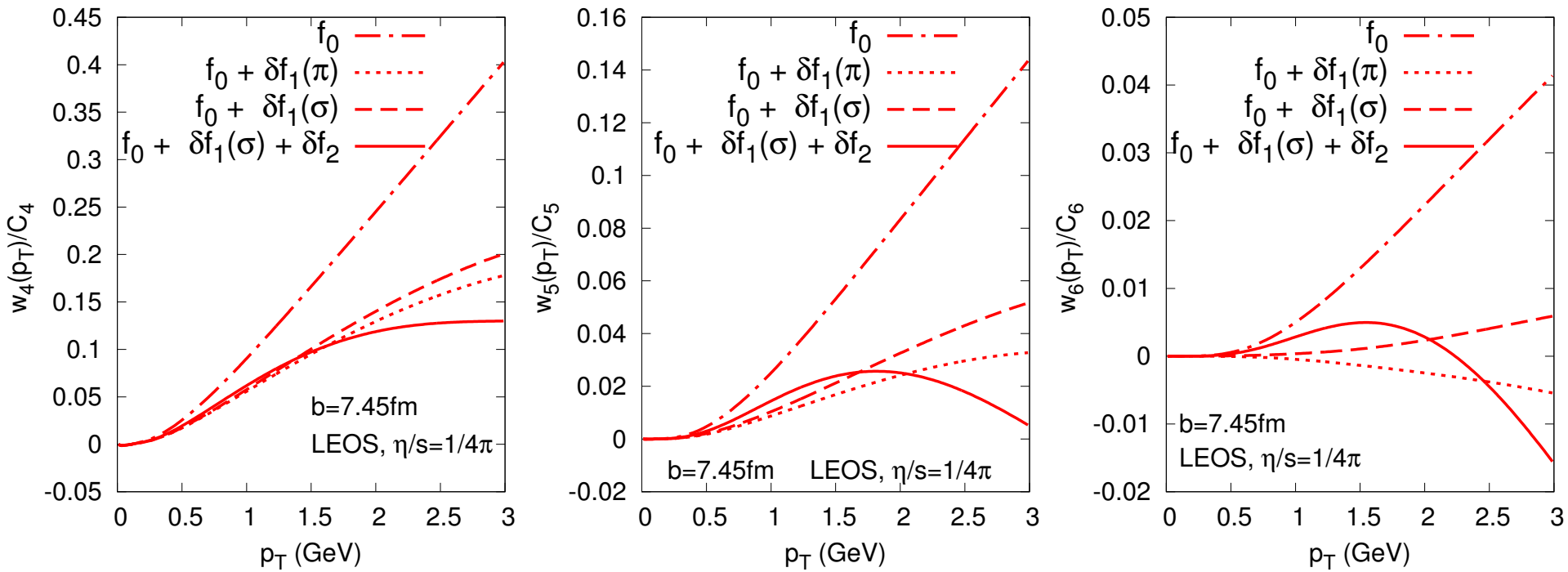
v_1 v_2 v_3



$\delta f_{(2)}$ and harmonic flow v_n

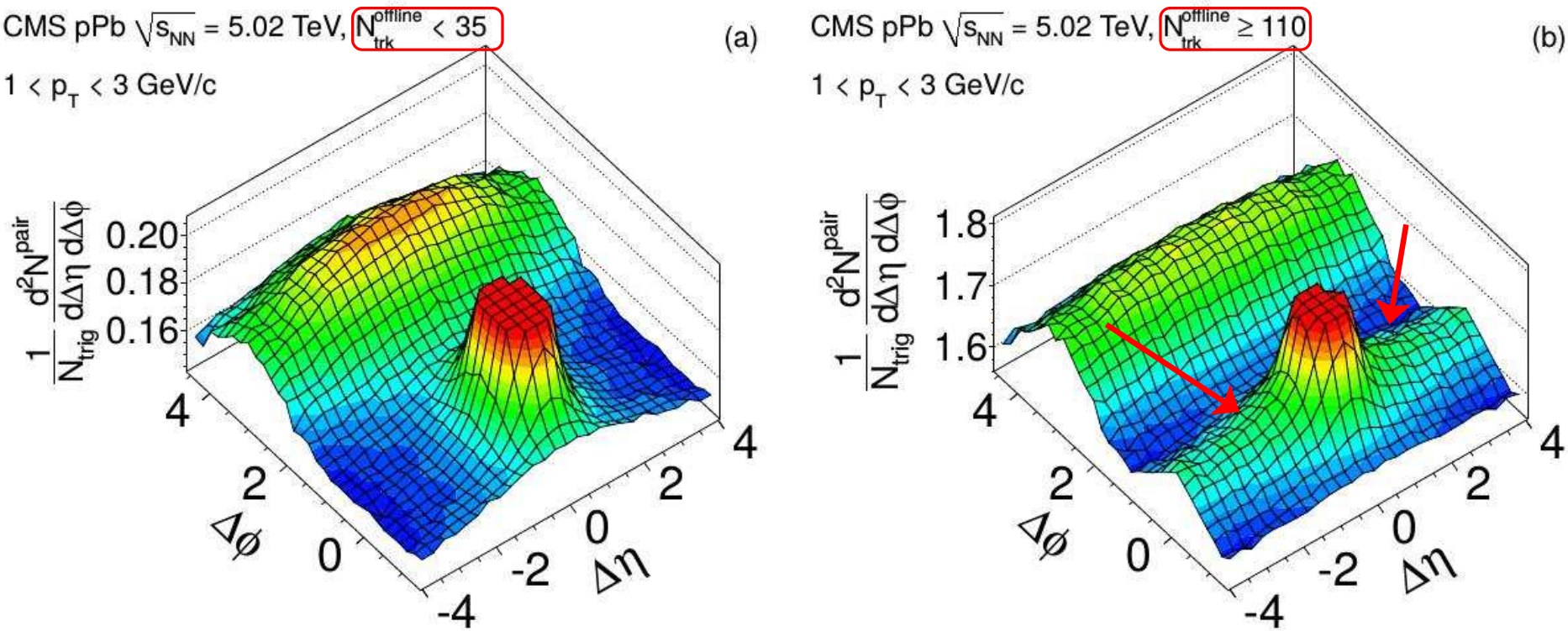
LHC PbPb:

v_4 v_5 v_6



Collectivity in small system?

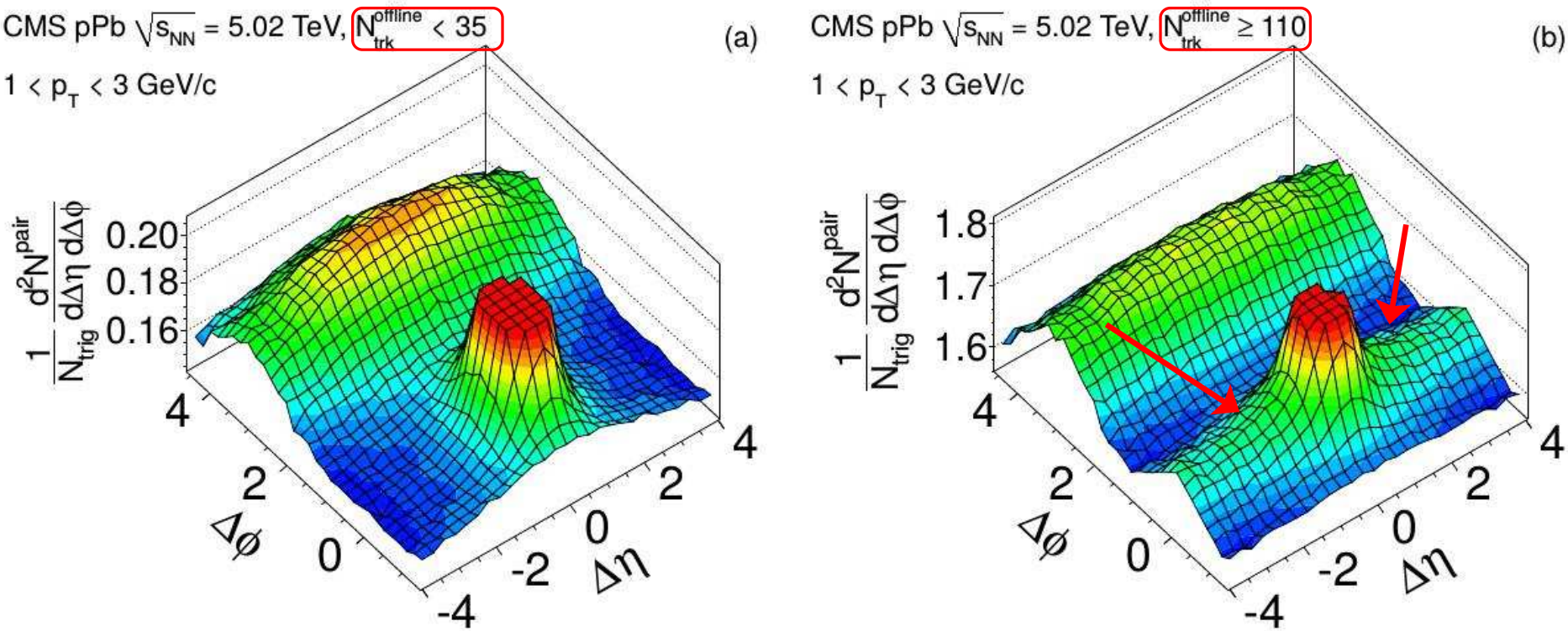
- CMS collaboration : proton-lead with $\sqrt{s} = 5.02$ TeV



(CMS collaboration, PLB718(2013) 795)

Collectivity in small system?

- CMS collaboration : proton-lead with $\sqrt{s} = 5.02$ TeV

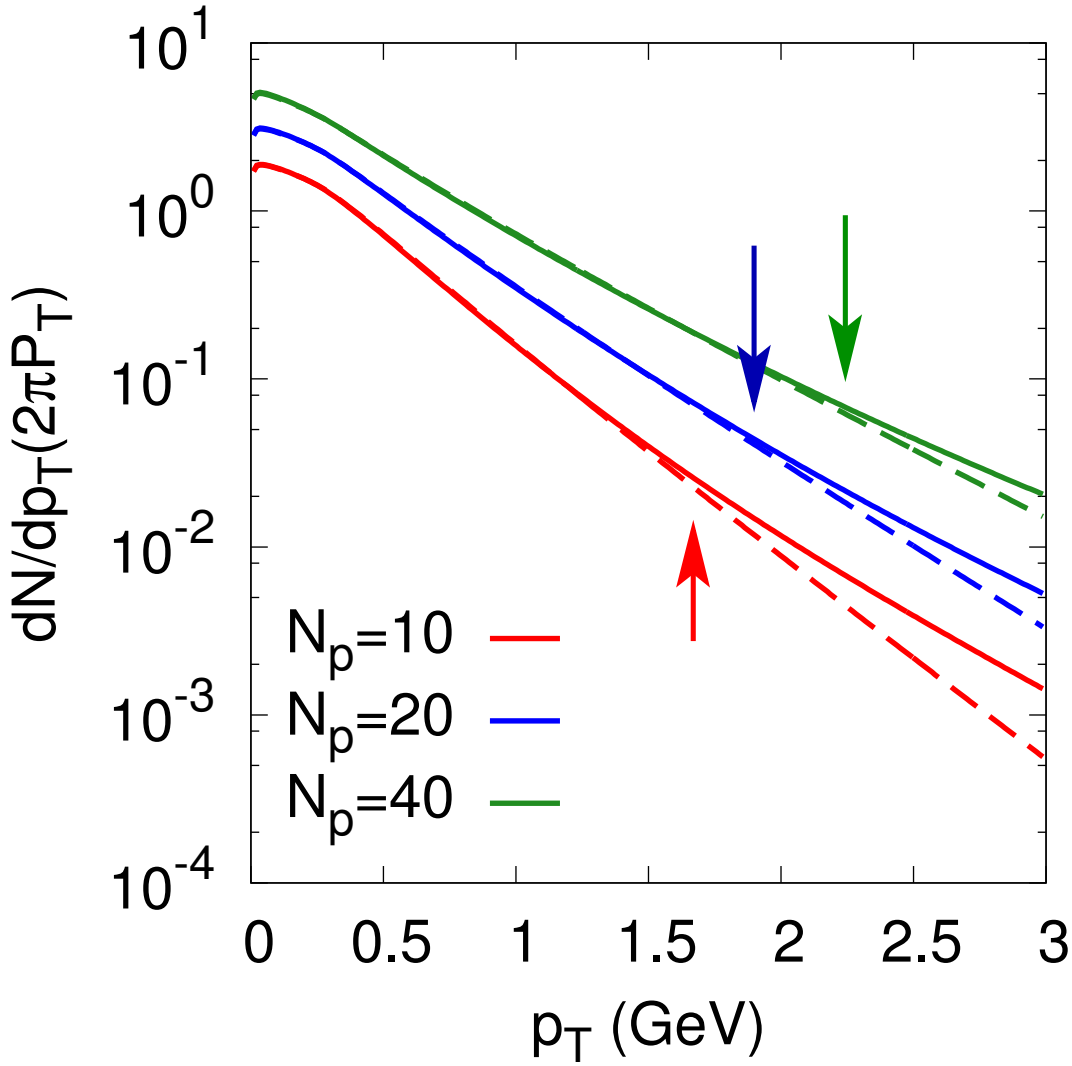


(CMS collaboration, PLB718(2013) 795)

Hydro. for small systems such as p-Pb?

Convergence of hydro. for small system

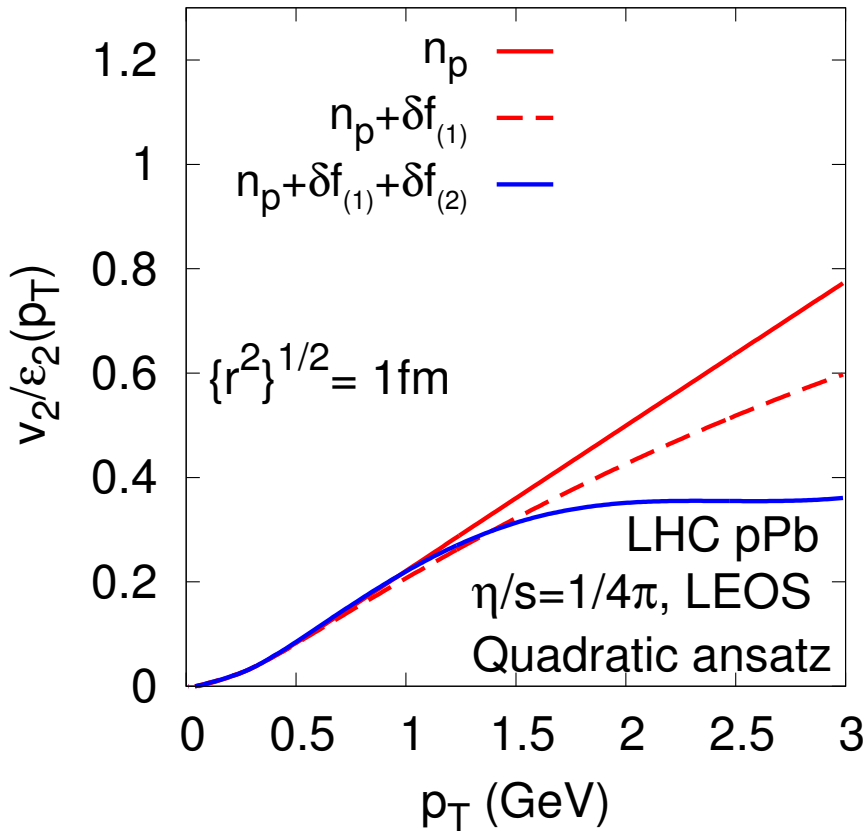
- p-Pb with fixed $\{r^2\}^{1/2}$=1 fm: dashed $\rightarrow n_p$, solid $\rightarrow n_p + \delta f_1 + \delta f_2$



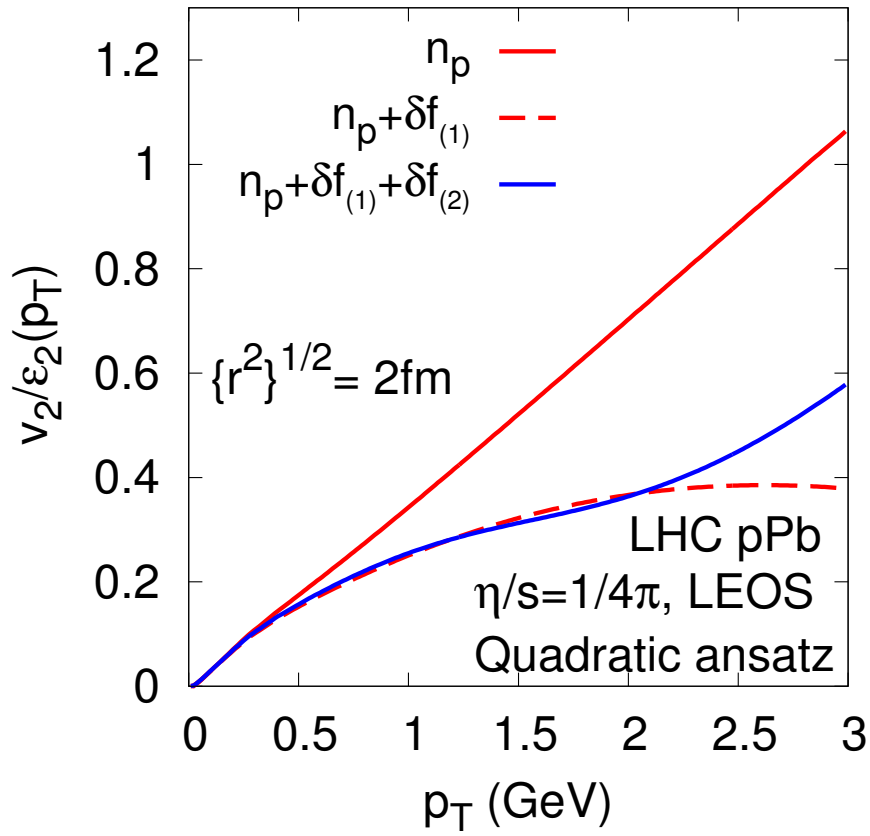
Convergence of hydro. for small system

- p-Pb with fixed N_p=20 ($N_{\rm track}^{\rm offline} \simeq 90$): elliptic flow v_2

$\{r^2\}^{1/2} = 1$ fm



$\{r^2\}^{1/2} = 2$ fm



Summary and conclusions

- We have derived a consistent form of δf for the 2nd order viscous hydro:

- General procedure from solving Boltzmann equation order by order.
- 2nd order transport coefficients are fixed – kinetic approach.
- Formulation can be extended to other EOM's, such as Israel-Stewart hydro.
- δf affects harmonic spectrum, especially for large n and p_T.
- Convergence of hydro. modelling for harmonic flow.

Back-up slides

Solving 2nd order viscous EOM – BRSSS

Equation of motion from the form of stress tensor, π as a dynamic variable

$$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau_\pi \left[{}^{\langle} D\pi^{\mu\nu\rangle} + \frac{4}{3}\pi^{\mu\nu}\nabla\cdot u \right] - \frac{\lambda_1}{\eta}\pi_\lambda^{\langle\,\mu}\pi^{\nu\lambda\rangle} - \frac{\lambda_2}{\eta}\pi_\lambda^{\langle\,\mu}\Omega^{\nu\lambda\rangle},$$