Anomalous fluid dynamics in even and odd dimensions

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- SP, Jian-Hua Gao, Qun Wang, Phys.Rev. D83 (2011) 094017
- Jian-Hua Gao, Zuo-Tang Liang, Shi Pu, Qun Wang, Xin-Nian Wang, arXiv:1203.0725

July 12, 2012

Outline

- Introduction
- Anomalous fluid in 3+1 dim
  - Chiral Magnetic Effect (CME)
  - Chiral Vortical Effect (CVE)
- Anomalous fluid in 2+1 dim
  - Hall conductivity
- Summary

Relativistic fluid dynamics

- Fluid dynamic is an effective theory for any interacting theory at finite temperature in long wavelength and low frequency limit.
- L.D. Landau (1953) first proposed to apply fluid dynamics to describe the hadronic fireballs.
- “RHIC physicists sever up the prefect liquid.”

Conservation equations

- Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \ ,$$

Energy-momentum tensor

- Charge current conservation

$$\partial_\mu j^\mu = 0 \ ,$$

Charge current

Gradient expansion

- Knudsen number

$$K \equiv \ell_{micro}/\ell_{macro} \sim l_{mfp}\partial_\mu$$

- Leading order

Ideal fluid dynamics

1st order

- Relativistic Navire-Stokes (NS) equations

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \,, \\
j^\mu &= n u^\mu + \nu^\mu \,, \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu
\end{aligned}
$$

ϵ	P	u^μ	Π
energy density	pressure	velocity	bulk viscous pressure
$\pi^{\mu\nu}$	n	ν^μ	
shear stress tensor	number density	diffusion current	

2nd order theory

- Open question

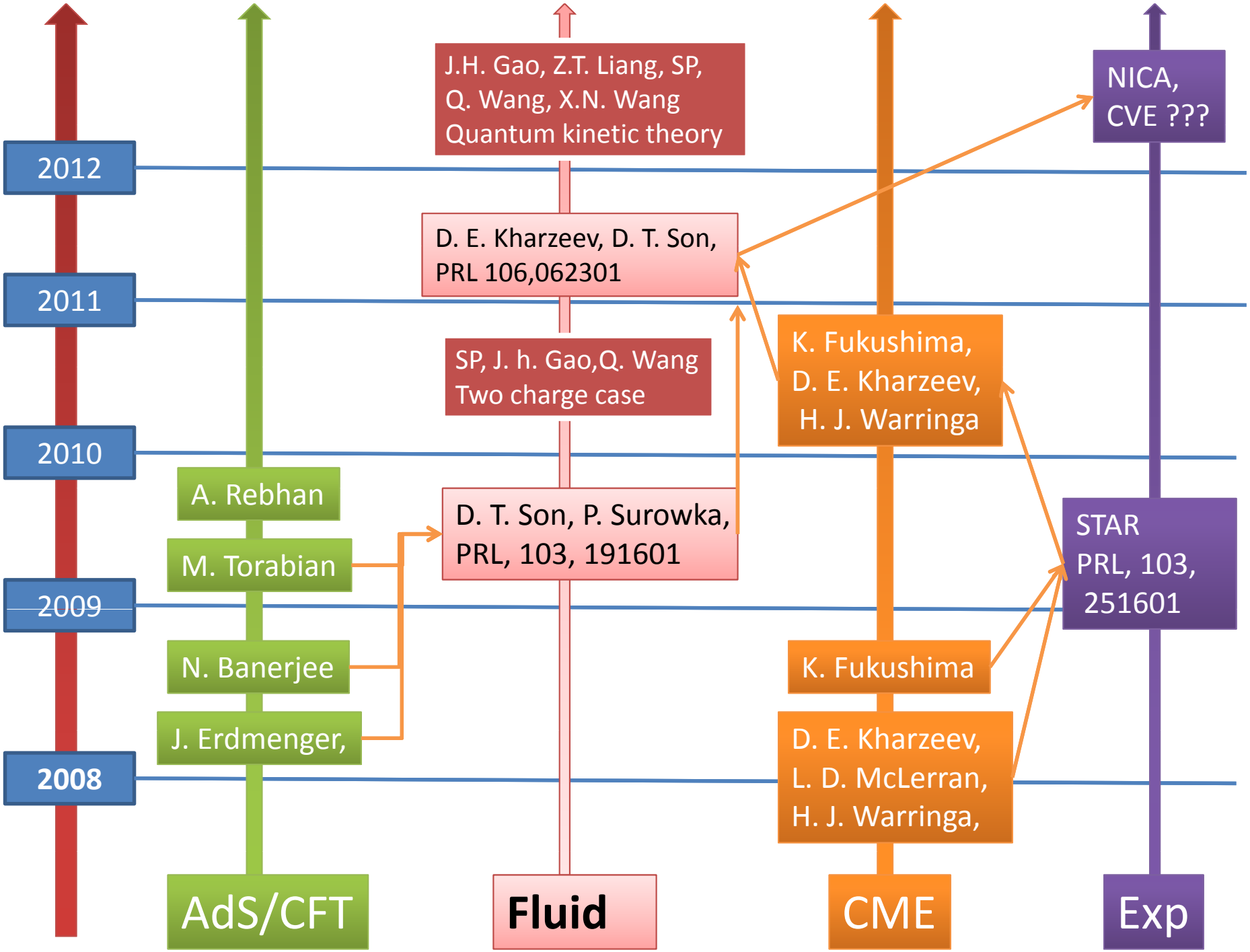
Candidates

- Israel-Stewart (IS) equations,
  - simplest version (IS 1979),
  - complete version (D. Rischke 2008, 2010),
- Conformal fluid (D.T. Son 2007),
  - includes the gravity, no bulk viscosity,
- Others,
  - memory function effect (T. Koide 2008, etc.)
  - extended thermodynamics (T. Koide 2008, etc.)
  - ...

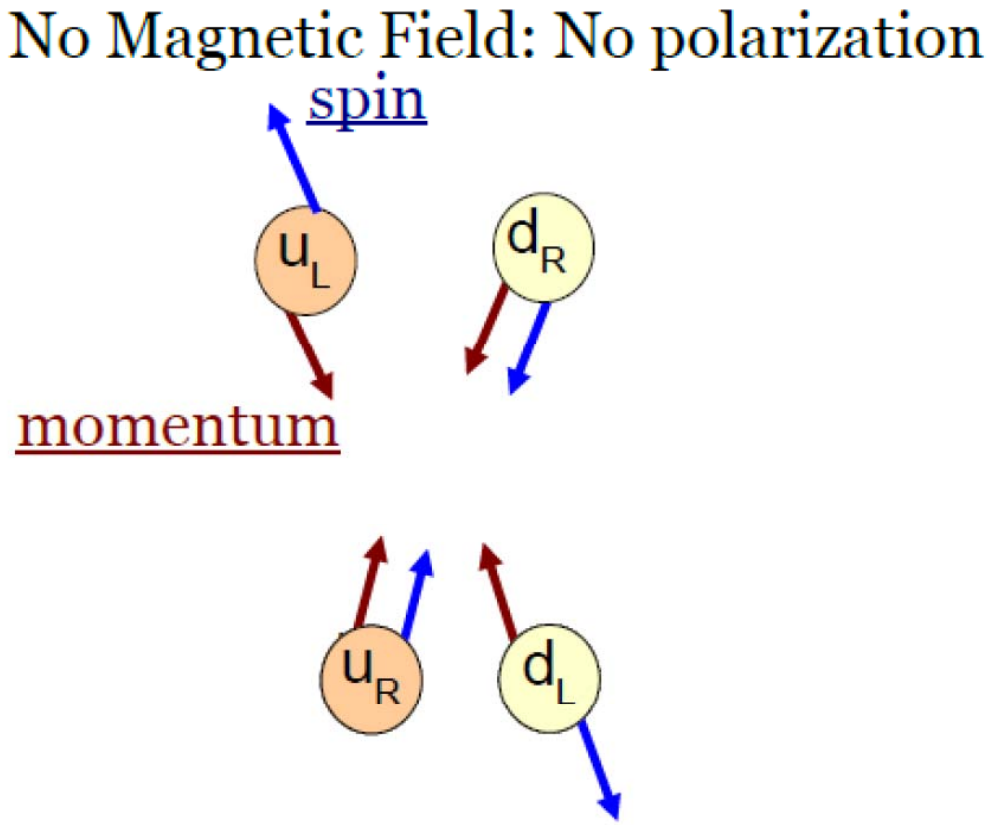
$$
\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \,, \\
j^\mu &= n u^\mu + \nu^\mu \,,
\end{aligned}
$$

- Why can these simple expression describe a very complicated system?
- We only describe the collective motions of the fluid cells.
- Can we use these to describe all relativistic system in “some” limit?

Anomalous fluid dynamics in dim=3+1


2012
2011
2010
2009
2008
AdS/CFT
Fluid
CME
Exp
A. Rebhan
M. Torabian
N. Banerjee
J. Erdmenger,
J.H. Gao, Z.T. Liang, SP, Q. Wang, X.N. Wang Quantum kinetic theory
D. E. Kharzeev, D. T. Son, PRL 106,062301
SP, J. h. Gao,Q. Wang Two charge case
D. T. Son, P. Surowka, PRL, 103, 191601
K. Fukushima, D. E. Kharzeev, H. J. Warringa
K. Fukushima
D. E. Kharzeev, L. D. McLerran, H. J. Warringa,
NICA, CVE ???
STAR PRL, 103, 251601

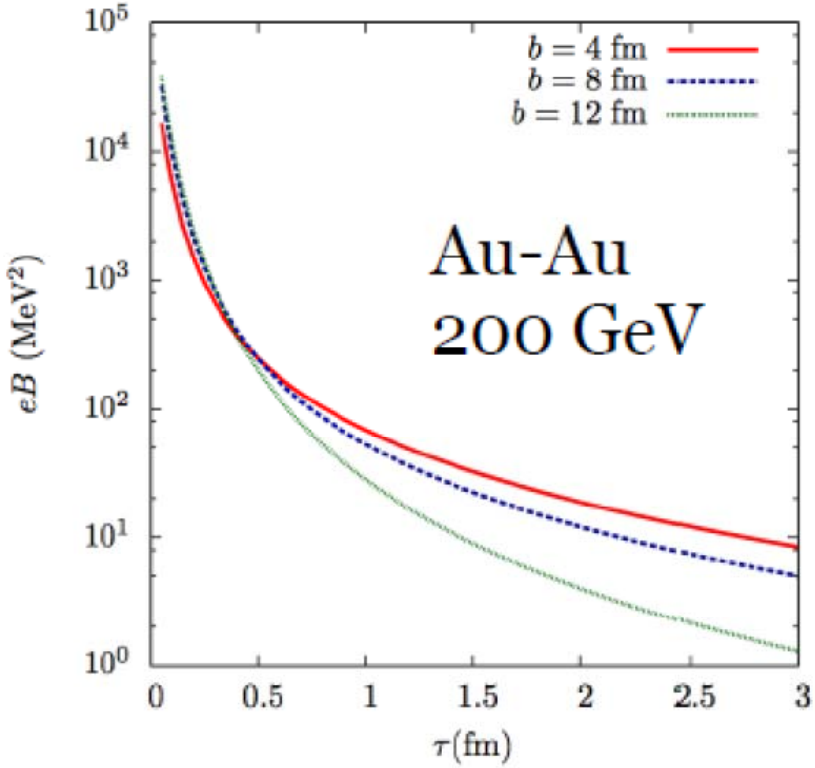
Helicity for massless fermions



For massless ferimons, the helicity is fixed.

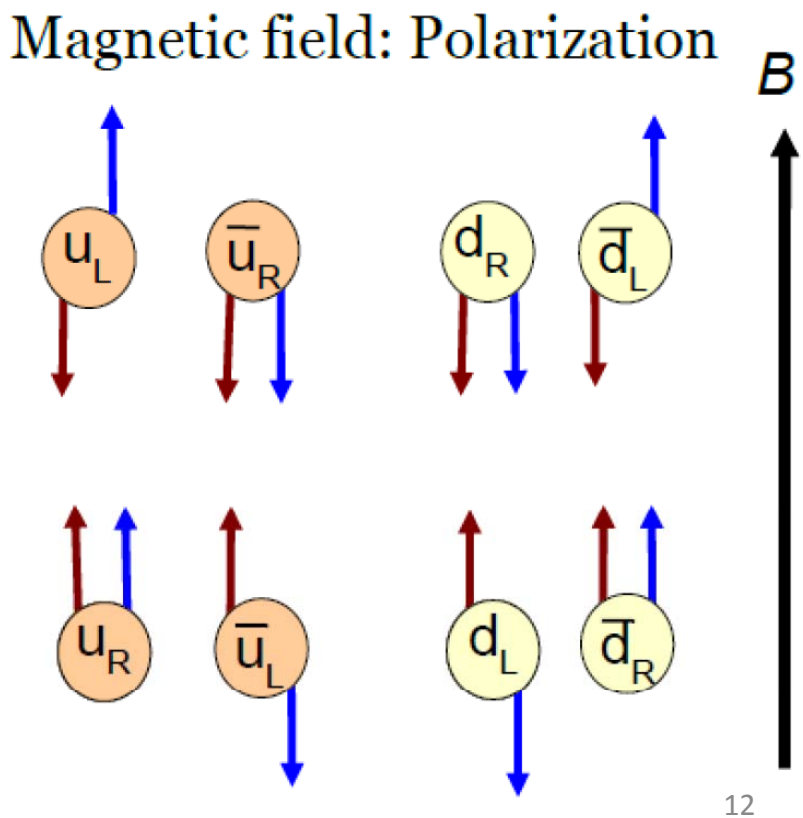
Polarization

In the non-central Au-Au collisions, there are strong magnetic fields.

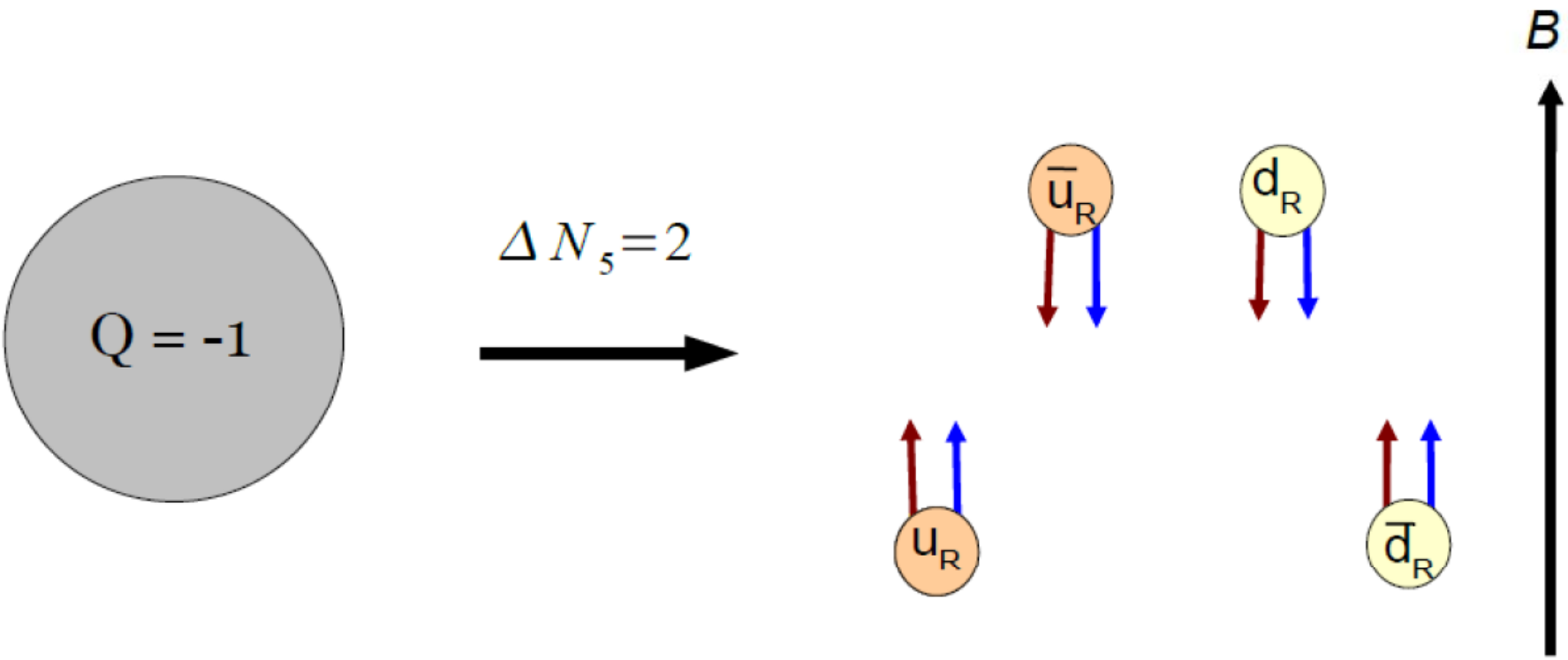


Pancake approximation
Kharzeev, McLerran & HJW ('o8)

But if number of left- and right-handed quarks are the same, there will be no special effect.



Induced electromagnetic current



Q < -1: Positively charged particles move parallel to magnetic field, negatively charged antiparallel

... = Electromagnetic Current

- How to describe this effect by the standard fluid dynamics?

$$
\begin{aligned}
T^{\mu\nu} &= \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \,, \\
j^\mu &= n u^\mu + \nu^\mu \,,
\end{aligned}
$$

CME and CVE

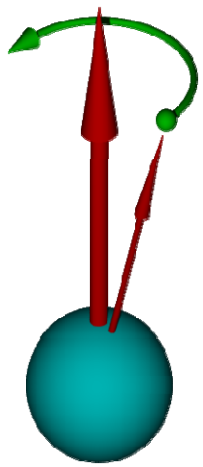
$$j^\mu = j^\mu_{normal} + \xi\omega^\mu + \xi_B B^\mu ,$$

Chiral Vortical Effect (CVE)

Chiral Magnetic Effect (CME)

$$\omega^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

Vorticity
Effective magnetic field
(Larmor precession)



Single charge anomalous fluid

D.T. Son et. al (2009, 2010) considered the fluid dynamics with the triangle anomalies

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu}_{\ \lambda} j^\lambda \ , \\ \partial_\mu j^\mu &= CE \cdot B \ . \end{aligned} \qquad (8)$$

C: anomalous constant

The entropy production rate is given by

$$\partial_\mu s^\mu = \frac{1}{T}(\Pi^2 + \pi^2 + \Delta^{\mu\nu}\nu_\mu\nu_\nu) - C\frac{\mu}{T}E\cdot B \qquad (9)$$

Chemical potential

Temperature

If $E \cdot B$ is large enough, $\partial_\mu s^\mu < 0$.

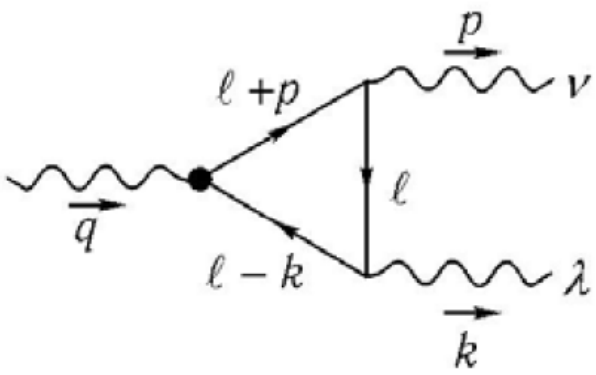
Single charge case

In order to satisfy the 2nd law of thermal dynamics, we have to modify

$$
\begin{aligned}
s^\mu &= s^\mu_{normal} + D\omega^\mu + D_B B^\mu \,, \\
j^\mu &= j^\mu_{normal} + \xi\omega^\mu + \xi_B B^\mu \,, \quad (12)
\end{aligned}
$$

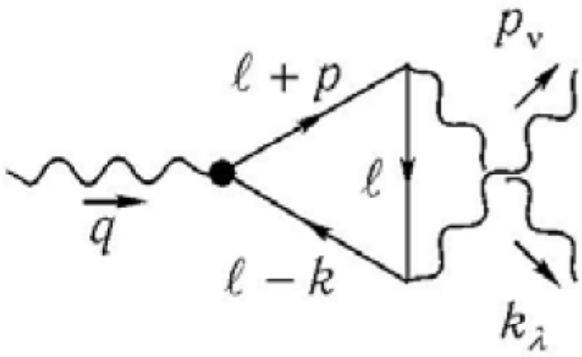
$$
\begin{aligned}
\xi &= C\mu^2 \left(1 - \frac{2}{3}\frac{n\mu}{\epsilon + P}\right) \,, \\
\xi_B &= C\mu \left(1 - \frac{1}{2}\frac{n\mu}{\epsilon + P}\right) \,.
\end{aligned}
$$

Chiral anomalies in dim=3+1



For massless Fermions, one finds that

$$\begin{aligned} \partial_\mu j^\mu &\equiv \partial_\mu (j_L^\mu + j_R^\mu) = 0 \ , \\ \partial_\mu j_5^\mu &\equiv \partial_\mu (j_R^\mu - j_L^\mu) \\ &= C F^{\mu\nu} \widetilde{F}_{\mu\nu} \ , \end{aligned} \tag{7}$$



where $j_L^\mu = \overline{\psi}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right)\psi$, $j_R^\mu = \overline{\psi}\gamma^\mu \left(\frac{1+\gamma^5}{2}\right)\psi$ and C is a constant.

Two charge case

$$\begin{aligned}
\partial_\mu T^{\mu\nu} &= F^{\nu}_{\lambda} j^{\lambda} \,, \\
\partial_\mu j^{\mu} &= 0 \,, \\
\partial_\mu j_5^{\mu} &= CE \cdot B \,,
\end{aligned}$$

where $j^\mu = j_R^\mu + j_L^\mu$ and $j_5^\mu = j_R^\mu - j_L^\mu$. We assumed that

$$\begin{aligned}
j^{\mu} &= j^{\mu}_{normal} + \xi \omega^{\mu} + \xi_B B^{\mu} \,, \\
j_5^{\mu} &= j^{\mu 5}_{normal} + \xi_A \omega^{\mu} + \xi_{AB} B^{\mu} \,.
\end{aligned}$$

- SP, J. Gao and Q. Wang, Phys.Rev. D83 (2011) 094017

To keep $\partial_\mu s^\mu \geq 0$, we get

$$\xi = -2C\mu\mu_A\left(1 - \frac{n\mu}{\epsilon + P}\right) ,$$

$$\xi_A = -C\mu^2\left(1 - \frac{2n_A\mu_A}{\epsilon + P}\right) ,$$

$$\xi_B = -C\mu_A\left(1 - \frac{n\mu}{\epsilon + P}\right) ,$$

$$\xi_{AB} = -C\mu\left(1 - \frac{n_A\mu_A}{\epsilon + P}\right)$$

- In small $\mu_A = \mu_R - \mu_L$ limit, the solution (16) is identical to the results from CME (Fukushima 2008, Kharzeev 2010).
- This results are also independently obtained by other groups (Sadofyev 2010, Amad 2011)

Boltzmann Eq.

- We also use the classical Boltzmann equations to investigate the quantum anomalous fluid.

- However, we have to use entropy principle to obtain the new coefficients even if we have used the “kinetic theory”.

- Classical Boltzmann equation cannot be the kinetic theory of anomalous fluid.

- CME and CVE: pure quantum effect
- Boltzmann equation: semi-classical theory

$$\partial_\mu j_5^\mu = CE \cdot B \ ,$$

$$j^\mu = j^\mu_{normal} + \xi \omega^\mu + \xi_B B^\mu \ ,$$

$$j_5^\mu = j^{\mu 5}_{normal} + \xi_A \omega^\mu + \xi_{AB} B^\mu \ .$$

- We have put everything by hand or by using entropy principle.
- How to obtain the triangle anomaly from kinetic theory? How to describe the CME and CVE in a kinetic theory?

Quantum kinetic theory

- Gauge invariant Wigner function

  quantum ferimon distribution function

$$\hat{W}_{\alpha\beta} = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \Psi_{\alpha\beta}(x,y) \ ,$$

$$\Psi_{\alpha\beta} \equiv \bar{\psi}_\beta(x+\frac{1}{2}y) \boxed{\mathcal{P}e^{-iy^\mu \int_0^1 ds A_\mu(x-\frac{1}{2}y+sy)}} \psi_\alpha(x-\frac{1}{2}y)$$

Gauge link

- J.H. Gao, Z.T. Liang, SP, Q. Wang, X.N. Wang, arXiv:1203.0725

“Dirac” equation

- In a homogenous external electromagnetic field,

$$(\gamma \cdot K - m)W = 0 \ ,$$

$$K_\mu = \frac{1}{2} i \partial^x_\mu + p_\mu - \frac{1}{2} F^{ext}_{\mu\nu} \partial^\nu_p = p_\mu + \frac{1}{2} i \nabla_\mu \ ,$$

$$\nabla_\mu \equiv \partial^x_\mu - F^{ext}_{\mu\nu} \partial^\nu_p \ .$$

- Compared with no gauge link case, we can easily find

Energy-momentum tensor

$$T_{\mu\nu} = -\frac{i}{2}\left\langle : \bar{\psi}[\gamma_\nu \overleftarrow{\partial_\mu} - \gamma_\nu \partial_\mu]\psi : \right\rangle = \mathrm{tr} \int d^4p p_\mu \gamma_\nu W$$

Charge current

$$j^\mu = e\int d^4p \gamma^\mu_{\beta\alpha} \left\langle : \psi\bar{\psi} : \right\rangle_{\alpha\beta} = e\int d^4p \mathrm{tr}\ (\gamma^\mu W)$$

Chiral current (axial current)

$$j^{\mu 5} = \int d^4p \mathrm{Tr}\ [\gamma^\mu \gamma^5 W]$$

- If we can solve the "Dirac" equation for Wigner functions, we can get the everything.

Assumption

- In massless and homogenous external field limit, we ignore the interaction between particles.
- Gradient expansion

$$W = W_0 + W_1 + O(\partial^2),$$

- The leading order should be correspondent to the ideal fluid

$$j_0^\mu = \int \frac{d^3p}{(2\pi)^3} p^\mu f,$$

f : Fermi-Dirac distribution

The leading order

- In homogenous (constant) external field limit, we ignore the interaction between particles.
- All dissipative terms vanish.

$$\Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}\Delta_{\alpha\beta}\Delta^{\rho\sigma}\partial_\rho u_\sigma\right) = 0,$$

$$T\Delta^{\mu\nu}\partial_\nu\frac{\mu}{T} + QE^\mu = 0, \quad u\cdot\partial u^\mu - \Delta^{\mu\nu}\partial_\nu \ln T = 0,$$

$$\partial_\mu\frac{\mu_5}{T} = 0, u^\mu\partial_\mu\frac{\mu}{T} = 0, u\cdot\partial T + \frac{1}{3}T\Delta^{\rho\sigma}\partial_\rho u_\sigma = 0 \quad (13)$$

- Equation of motion of ideal fluid

1st order

$$j^\mu = nu^\mu + \xi\omega^\mu + \xi_B B^\mu,$$

$$j_5^\mu = n_5 u^\mu + \xi_5\omega^\mu + \xi_{B5} B^\mu,$$

These results are consistent with Landsteiner, Megias and Pena-Benitez's work (Phys. Rev. Lett. 107, 021601)

$$\xi = \frac{1}{\pi^2}\mu\mu_5,$$

Q: charge

$$\xi_B = \frac{Q}{2\pi^2}\mu_5,$$

$$\xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right),$$

$$\xi_{B5} = \frac{Q}{2\pi^2}\mu.$$

Multi-flavor case

$$\xi^{\text{baryon}} = \frac{N_c}{\pi^2}\mu\mu_5, \ \xi_B^{\text{baryon}} = \frac{N_c}{6\pi^2}\mu_5 \sum_f Q_f,$$

Baryonic current

$$\xi^{\text{EM}} = \frac{N_c}{\pi^2}\mu\mu_5 \sum_f Q_f, \ \xi_B^{\text{EM}} = \frac{N_c}{2\pi^2}\mu_5 \sum_f Q_f^2.$$

Electromagnetic current

In three flavor case, $\sum_f Q_f = 0,$

$$\xi_B^{\text{baryon}} = \xi^{\text{EM}} = 0,$$

Nc: fundamental color charges

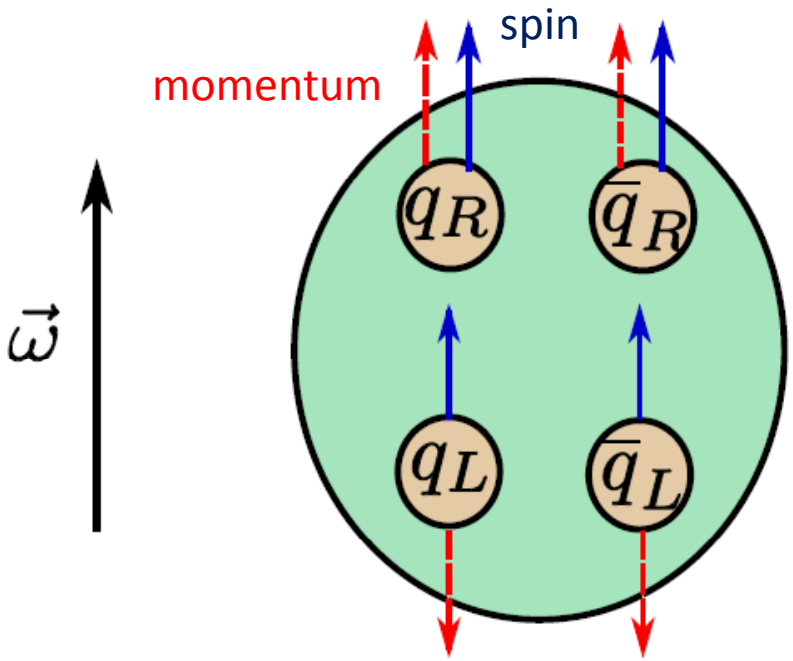
- CME/CVE dominates the electromagnetic/baryonic current.
- It is consistent with the result of D. E. Kharzeev and D. T. Son (Phys. Rev. Lett. 106,062301)

Local polarization effect (LPE)

$$j_5^\mu = n_5 u^\mu + \xi_5 \omega^\mu + \xi_{B5} B^\mu,$$

$$\xi_5 = N_c \left[\boxed{\frac{1}{6} T^2} + \frac{1}{2\pi^2}(\mu^2 + \mu_5^2) \right],$$

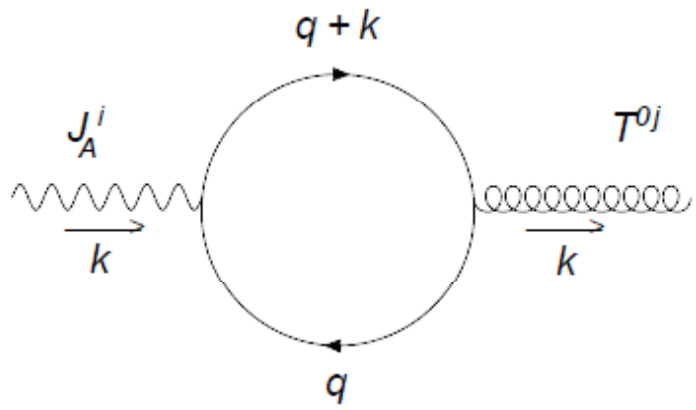
$$j_5^\mu \equiv (j_R^\mu - j_L^\mu)$$



- It is similar to the global one.
- The LPE is protected in both high and low energy heavy-ion collisions at either low baryonic chemical potentials and high temperatures or vice versa.

What is new in theoretical field?

$$
\begin{aligned}
\xi &= \frac{1}{\pi^2}\mu\mu_5, \\
\xi_B &= \frac{Q}{2\pi^2}\mu_5, \\
\xi_5 &= \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2+\mu_5^2\right), \\
\xi_{B5} &= \frac{Q}{2\pi^2}\mu.
\end{aligned}
$$



Landsteiner, Megias and Pena-Benitez's work (Phys.Rev. Lett. 107, 021601)

- We obtain everything without the entropy principle!
- The Wigner function can describe particle distribution in a quantum system.

Differential equations

- We obtain the triangle anomaly by kinetic theory!

$$\partial_\mu T^{\mu\nu} = Q F^{\nu\rho} j_\rho,$$

$$\partial_\mu j^\mu = 0,$$

Triangle anomaly

$$\partial_\mu j_5^\mu = \boxed{-\frac{Q^2}{16\pi^2}\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}}$$

Regularized chiral current

$$j^{\mu 5} = \int d^4 p \mathrm{Tr}\ [\gamma^\mu \gamma^5 W]$$

$$\hat{W}_{\alpha\beta} = \int \frac{d^4 y}{(2\pi)^4} e^{-ip\cdot y} \Psi_{\alpha\beta}(x, y)\ ,$$

$$\Psi_{\alpha\beta} \equiv \bar{\psi}_\beta(x + \frac{1}{2}y) \mathcal{P} e^{-iy^\mu \int_0^1 ds A_\mu(x - \frac{1}{2}y + sy)} \psi_\alpha(x - \frac{1}{2}y)$$

Gauge link

$$\begin{aligned} j^{\mu 5} &= \lim_{\epsilon \to 0} \overline{\psi}(x + \frac{1}{2}\epsilon)\gamma^\mu \gamma^5 \boxed{\mathcal{P} e^{-i\epsilon^\mu \int_0^1 ds A_\mu(x - \frac{1}{2}\epsilon + s\epsilon)}} \psi(x - \frac{1}{2}\epsilon) \\ &= \lim_{\epsilon \to 0} \overline{\psi}(x + \frac{1}{2}\epsilon)\gamma^\mu \gamma^5 \boxed{\mathcal{P} \exp[-i \int_{x-\epsilon/2}^{x+\epsilon/2} dz \cdot A(z)]} \psi(x - \frac{1}{2}\epsilon), \end{aligned}$$

We define a Regularized chiral current with gauge link, which is as the same as it in the quantum field theory.

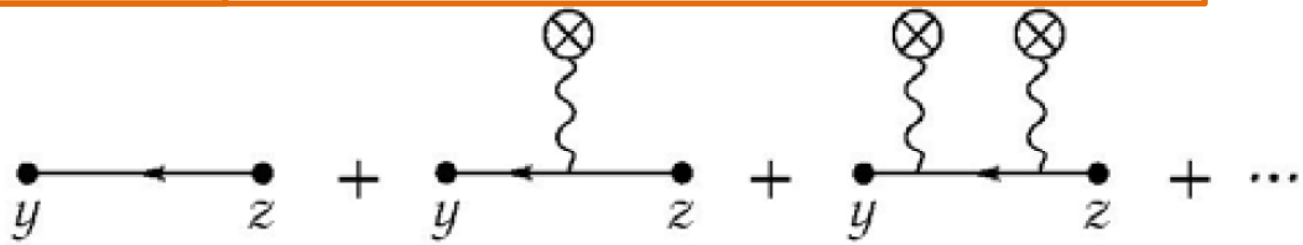
Triangle anomaly in quantum field theory

- The key to obtain triangle anomaly with an **external field** is to regularize the two point Green function.

Contribution from gauge link

$$\partial_\mu j^{\mu 5} = \underset{\epsilon \to 0}{\text{symm}} \lim \left\{ \bar{\psi}(x+\tfrac{\epsilon}{2}) \left[-ie\gamma^\mu \epsilon^\nu (\partial_\mu A_\nu - \partial_\nu A_\mu) \right] \gamma^5 \psi(x-\tfrac{\epsilon}{2}) \right\}$$

$$\overbrace{\psi(y)\bar{\psi}(z)} =$$



traceless, ~ 0

$$2e\epsilon^{\alpha\beta\mu\gamma} F_{\alpha\beta}(x) \left(\frac{-i}{8\pi^2} \frac{\epsilon_\gamma}{\epsilon^2} \right)$$

Finite

High order of ϵ ~ vanishing

- However, we do not regularize the fields! We just solve a “Dirac” equations in gradient expansion!

$$(\gamma \cdot K - m)W = 0 \ ,$$

$$j^{\mu 5} = \int d^4 p \mathrm{Tr} \ [\gamma^\mu \gamma^5 W]$$

- Our system is
  - Massless fermion in an homogenous external field
  - No interaction between particles
- We only consider two reasonable assumptions:
  - Gradient expansion
  - 0th order is ideal fluid
- We do not
  - Regularized current
  - Calculate the loop diagrams
- However, we have obtained the same results as
  - Regularization from quantum field theory
  - Loop calculations at finite temperature

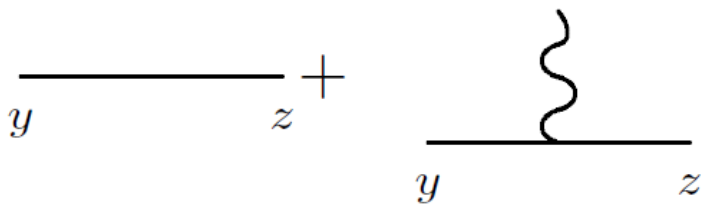
- Why can we use so simple assumptions to obtain the triangle anomaly without regularization and loop calculations?

- There must be some deep physical “things” in the anomalous fluid!

- In my opinion, the reason might be the Wigner function obeys the Dirac equation in an external field.

$$(i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu)\psi = 0,$$

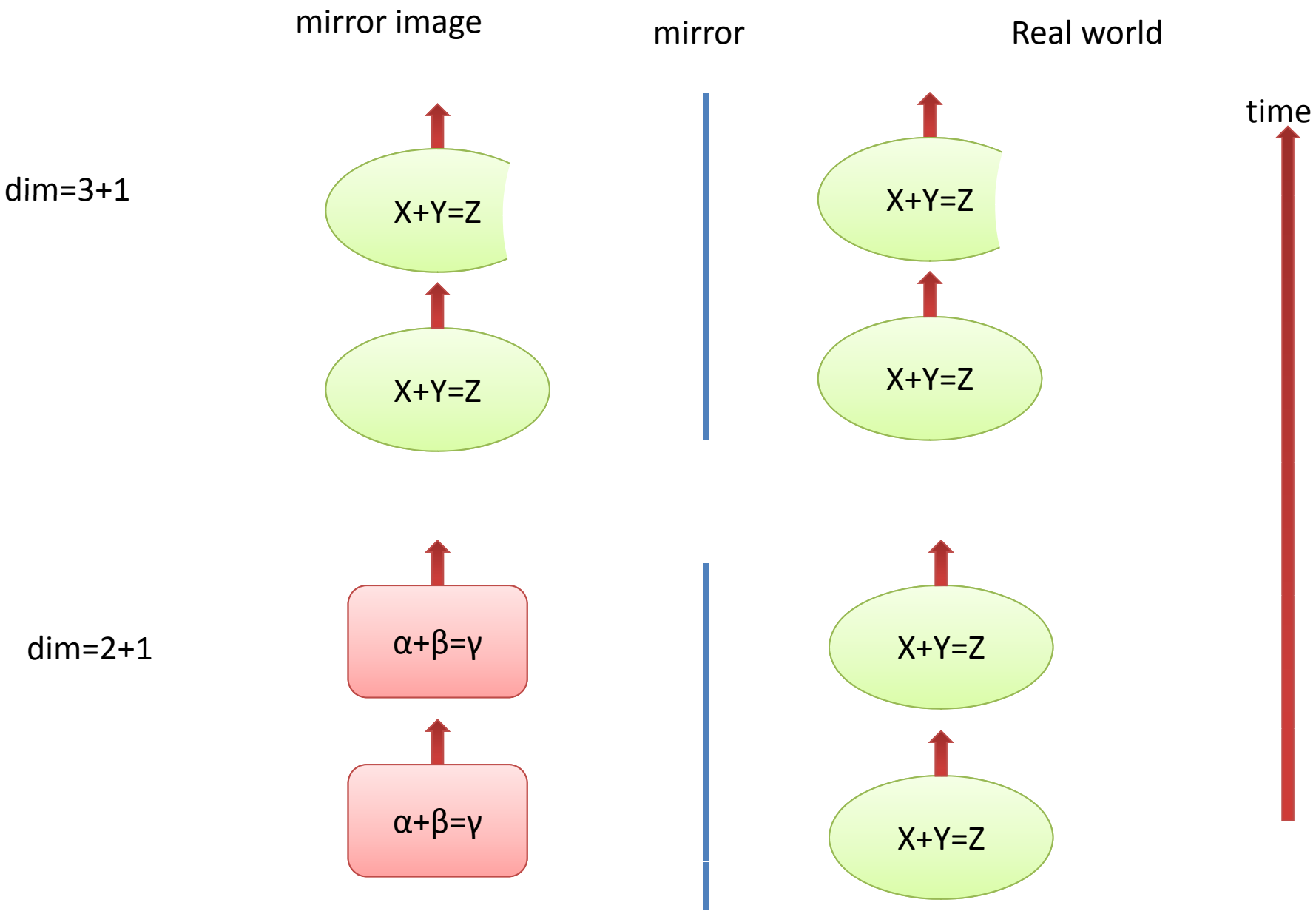
$$\begin{aligned}
\lim_{y\to z} \overline{\psi}(y)\psi(z) &= \frac{i}{i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu} \\
&= \frac{i}{\gamma^\mu \partial_\mu} - i\frac{i}{\gamma^\mu \partial_\mu} e\gamma^\mu A_\mu \frac{i}{\gamma^\mu \partial_\mu} + \ldots \\
&= \quad + \quad + \ldots,
\end{aligned}$$



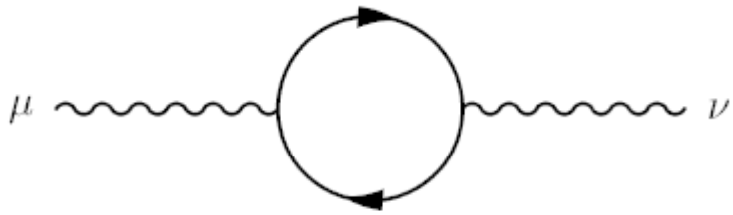
Anomalous fluid dynamics in dim=2+1

- The differences in

	dim=3+1	dim=2+1
mass	massless	massive, massless
Anomaly	Chiral anomaly	Parity anomaly, mass term breaks the parity
Current conservation	No $\partial_\mu j^{5\mu} = -CE \cdot B,$	Yes $\partial_\mu j^{\mu} = 0,$
Current	Normal expression	$j_\mu = \frac{e}{8\pi}\frac{m}{\lvert m\rvert}\epsilon_{\mu\alpha\beta}F^{\alpha\beta},$
Effect in experiment	CME and CVE	Hall effect


mirror image
mirror
Real world
time
dim=3+1
X+Y=Z
X+Y=Z
X+Y=Z
X+Y=Z
dim=2+1
α+β=γ
α+β=γ
X+Y=Z
X+Y=Z

Parity anomaly in Abelian case



Induced current $$j_\mu = \frac{e}{8\pi}\frac{m}{|m|}\epsilon_{\mu\alpha\beta}F^{\alpha\beta},$$

Hall effect $$j_x \propto E_y$$

* S. Deser, R. Jackiw and S. Templeton, Ann. Phys. 281, 409449 (2000)

Quantum kinetic theory

- We also obtain the Hall effect by the Wigner function.
- At zero temperature and large chemical potential limit, our result is the same as the one from loop calculations.

- Up to 1st order, the "complete" expression for fluid dynamics in dim=2+1 with parity violation is

$$T^{\mu\nu} = \epsilon_0 u^\mu u^\nu + \left(P_0 - \zeta \nabla_\alpha u^\alpha - \tilde{\chi}_B B - \tilde{\chi}_\Omega \Omega\right) \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu} \,,$$

$$J^\mu = \rho_0 u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T \,.$$

$$\Omega = -\epsilon^{\mu\nu\rho} u_\mu \nabla_\nu u_\rho, \qquad B = -\frac{1}{2} \epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho},$$

$$E^\mu = F^{\mu\nu} u_\nu, \qquad V^\mu = E^\mu - T \Delta^{\mu\nu} \nabla_\nu \frac{\mu}{T},$$

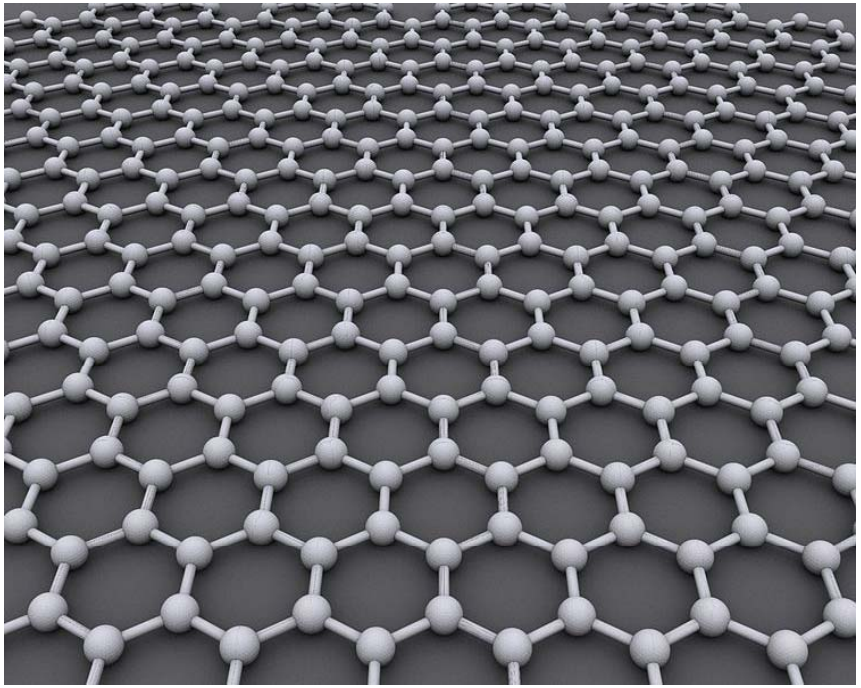
$$\Delta^{\mu\nu} = u^\mu u^\nu + g^{\mu\nu}, \qquad \sigma^{\mu\nu} = \Delta^{\mu\alpha} \Delta^{\nu\beta} \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - g_{\alpha\beta} \nabla_\lambda u^\lambda\right)$$

$$\tilde{E}^\mu = \epsilon^{\mu\nu\rho} u_\nu E_\rho \,, \qquad \tilde{V}^\mu = \epsilon^{\mu\nu\rho} u_\nu V_\rho \,,$$

$$\tilde{\sigma}^{\mu\nu} = \frac{1}{2} \left(\epsilon^{\mu\alpha\rho} u_\alpha \sigma_\rho{}^\nu + \epsilon^{\nu\alpha\rho} u_\alpha \sigma_\rho{}^\mu\right) \,.$$

* Kristan Jensen, Matthias Kaminskib, et.al, JHEP 1205 (2012) 102

Graphene



- The quasiparticles in graphene were 2+1 dimensional massless Dirac fermions!

- D. P. DiVincenzo, E. J. Mele, Phys. Rev. B 29, 1685–1694 (1984).
- K. S. Novoselov, et.al, Science 22 October 2004: **306** (5696), 666-669.

Summary

- We investigate the CME and CVE by quantum kinetic theory. Without any regularization and loop calculations, we obtain the triangle anomaly and some new transport coefficients.
- The local polarization effect is protected in both high and low energy heavy-ion collisions at either low baryonic chemical potentials and high temperatures or vice versa.
- Our method can also reproduce the Hall effect in 2+1 dimension system, i.e. graphene.

Thank you!

Appendix

The complete transport coefficients up to the second order of the Grad 14-moment expansion are

$$
\begin{aligned}
\pi^{\mu\nu} &= \pi^{\mu\nu}_{NS} - \tau_\pi \dot{\pi}^{<\mu\nu>} + 2 l_{\pi q} \nabla^{<\mu} q^{\nu>} \\
&\quad + 2\tau_{\pi q} q^{<\mu} \dot{u}^{\nu>} + 2\tau_\pi \pi_\lambda^{<\mu} \omega^{\nu>\lambda} - 2\eta \hat{\delta}_2 \pi^{\mu\nu} \theta \\
&\quad - 2\tau_\pi \pi_\lambda^{<\mu} \sigma^{\nu>\lambda} - 2\lambda_{\pi q} q^{<\mu} \nabla^{\nu>} \alpha + 2\lambda_{\pi\Pi} \Pi \sigma^{\mu\nu} \ , \\
q^\mu &= q^\mu_{NS} - \tau_q \Delta^{\mu\nu} \dot{q}_\nu + l_{q\Pi} \nabla^\mu \Pi - l_{q\pi} \Delta^{\mu\nu} \partial^\lambda \pi_{\nu\lambda} \\
&\quad \tau_{q\Pi} \Pi \dot{u}^\mu - \tau_{q\pi} \pi^{\mu\nu} \dot{u}_\nu + \tau_q \omega^{\mu\nu} q_\nu - \frac{\kappa}{\beta} \hat{\delta}_1 q^\mu \theta \\
&\quad - \lambda_{qq} \sigma^{\mu\nu} q_\nu + \lambda_{q\Pi} \Pi \nabla^\mu \alpha + \lambda_{q\pi} \pi^{\mu\nu} \nabla_\nu \alpha \ , \\
\Pi &= \Pi_{NS} - \tau_\Pi \dot{\Pi} - l_{\Pi q} \partial \cdot q \\
&\quad + \tau_{\Pi q} q \cdot \dot{u} - \zeta \hat{\delta}_0 \Pi \theta \\
&\quad + \lambda_{\Pi q} q \cdot \nabla \alpha + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \ ,
\end{aligned}
\tag{29}
$$

where the coefficients are the functions of the mass and the chemical potential of particles, $\dot{A} = \frac{d}{d\tau} A$ and $\pi^{\mu\nu}_{NS}$, q^μ_{NS}, Π_{NS} are given by the Navier-Stokes (NS) equations

$$
\pi^{\mu\nu}_{NS} = 2\eta \sigma^{\mu\nu} \ , \ q^\mu_{NS} = -\frac{\kappa}{\beta} \frac{n}{\beta(\epsilon + P)} \nabla^\mu \alpha \ , \ \Pi_{NS} = -\zeta \theta \ . \tag{30}
$$

Symmetry and entropy principle

- Symmetry
  - conservation law
  - C. P. T. invariance

$$x \rightarrow y$$

$$T_{xy} \sim (\partial_x u_y - \partial_y u_x)$$

- Entropic principle

$$T^{\mu\nu} \sim (P - u^\mu \partial_\mu T)\Delta^{\mu\nu}$$

Conclusions: P- and CP-odd effects in heavy ion collisions

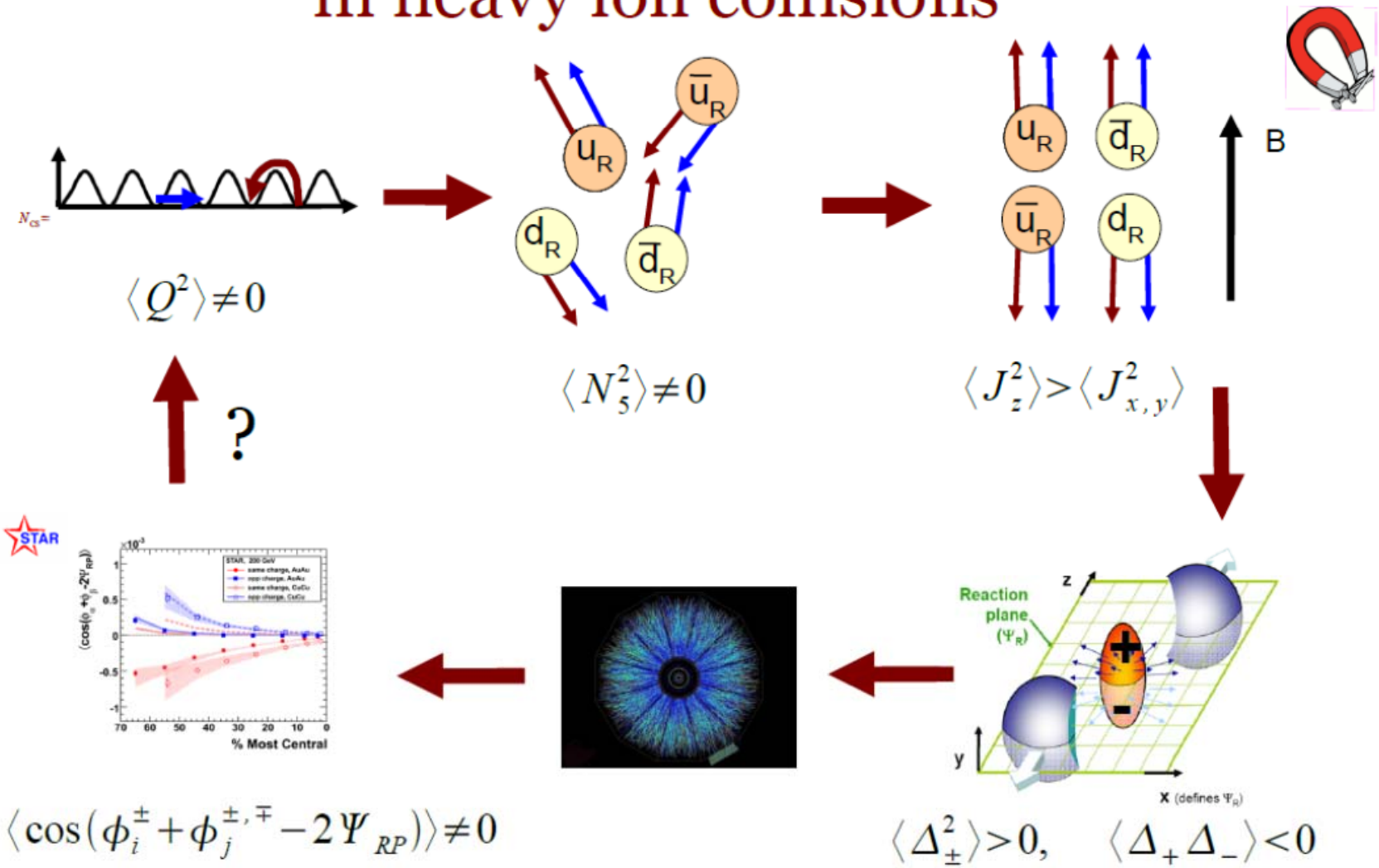
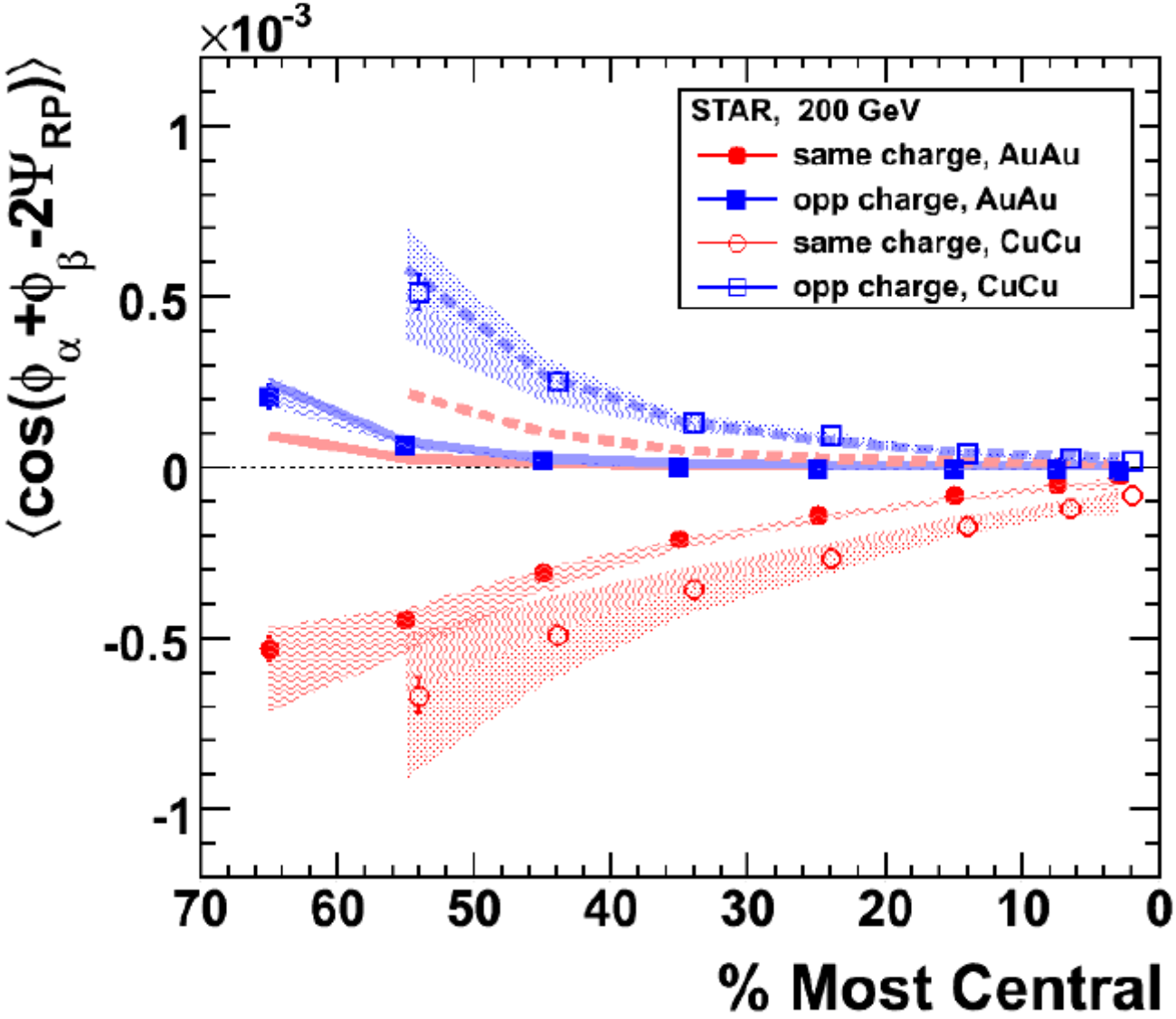


Figure: Copy from H. Warriga's talk on Hirschegg 2010.


×10^-3
⟨cos(φα+φβ-2ΨRP)⟩
1
0.5
0
-0.5
-1
70
60
50
40
30
20
10
0
% Most Central
STAR, 200 GeV
same charge, AuAu
opp charge, AuAu
same charge, CuCu
opp charge, CuCu

Simplicity

- We only consider the static solutions

$$
\begin{aligned}
u \cdot \partial u^{\mu} &= \Delta^{\mu\nu} \partial_{\nu} \ln T = 0, \\
u \cdot \partial T &= -\frac{1}{3} T \Delta^{\rho\sigma} \partial_{\rho} u_{\sigma} = 0.
\end{aligned}
$$

$$\longrightarrow \qquad \partial_{\mu} T = 0$$

Chiral vortaic effect and neutron asymmetries at NICA

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Abstract

We study the possibility of testing experimentally signatures of P-odd effects related with the vorticity of the medium. The Chiral Vortaic Effect is generalized to the case of conserved charges different from the electric one. In the case of baryonic charge and chemical potential such effect should manifest itself in neutron asymmetries at the NICA accelerator complex measured by the MPD detector. The required accuracy may be achieved in a few months of accelerator running. We also discuss polarization of the hyperons and P-odd correlations of particle momenta (handedness) as probes of vorticity.